Model Counting meets Distinct Elements

Circuits meet Data Streaming

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Joint work with Arnab Bhattacharyya, A. Pavan, and N.V. Vinodchandran

Corresponding publications: PODS-21 and 2023 CACM Research Highlights

Model Counting

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  - Boolean variables $X_1, X_2, \cdots X_n$
  - Formula φ over $X_1, X_2, \cdots X_n$
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Problem Compute (ε, δ) approximation of $|\mathsf{Sol}(\varphi)|$
Concern Number of NP Queries

Distinct Elements

- Given a stream $\mathbf{a} = a_1, a_2, \ldots a_m$ where $a_i \in \{0, 1\}^n$
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- Example $\mathbf{a} = 1, 2, 1, 1, 2, 1, 3, 5, 1, 2, 1, 3$
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  Concern Space Complexity

Hashing-Based Techniques

Model Counting (S83,GSS06,GHSS07,CMV13b,EGSS13b,CMV14,CDR15,CMV16,ZCSE16,AD16 KM18,ATD18,SM19,ABM20,SGM20)

Distinct Elements (FM85,AMS99,GT01,BKS02,BJKST02, CM03,CLKB04,PT07, TW12,SP09)

2-wise independent Hashing

- Let H be family of 2-wise independent hash functions mapping $\{0,1\}^n$ to $\{0,1\}^m$

$$\forall y_1, y_2 \in \{0,1\}^n, \alpha_1, \alpha_2 \in \{0,1\}^m, h \xleftarrow{R} H$$
$$\Pr[h(y_1) = \alpha_1] = \Pr[h(y_2) = \alpha_2] = \left(\frac{1}{2^m}\right)$$

$$\Pr[h(y_1) = \alpha_1 \wedge h(y_2) = \alpha_2] = \left(\frac{1}{2^m}\right)^2$$

2-wise independent Hash Functions

- Variables: $X_1, X_2, \cdots X_n$
- To construct $h : \{0,1\}^n \rightarrow \{0,1\}^m$, choose m random XORs
- Pick every X_i with prob. $\frac{1}{2}$ and XOR them
  - $X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2}$
  - Expected size of each XOR: $\frac{n}{2}$

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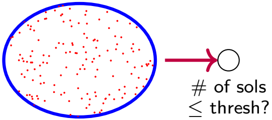
  - Expected size of each XOR: $\frac{n}{2}$
- To choose $\alpha \in \{0,1\}^m$, set every XOR equation to 0 or 1 randomly

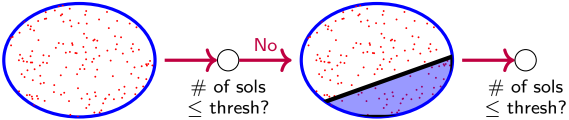
$$X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2} = 0 \qquad (Q_1)$$
$$X_2 \oplus X_5 \oplus X_6 \cdots \oplus X_{n-1} = 1 \qquad (Q_2)$$
$$\cdots \qquad (\cdots)$$
$$X_1 \oplus X_2 \oplus X_5 \cdots \oplus X_{n-2} = 1 \qquad (Q_m)$$

- Therefore, $h(X) = \alpha$ can be represented as $AX = b$

ApproxMC



ApproxMC



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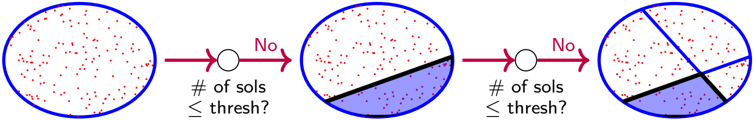

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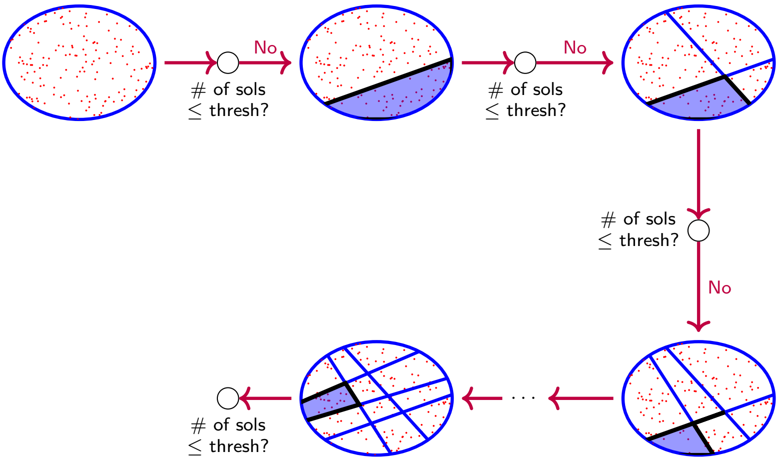

No
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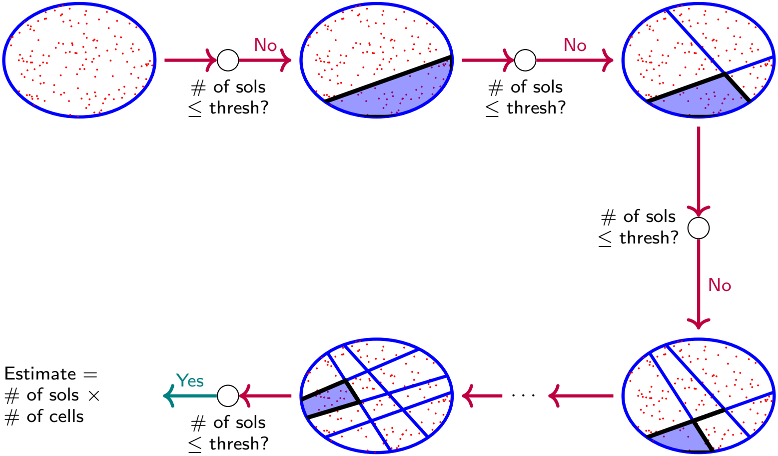
ApproxMC


No
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Yes
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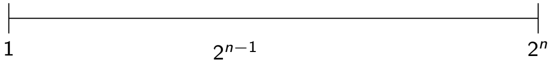
Distinct Elements



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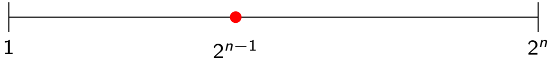

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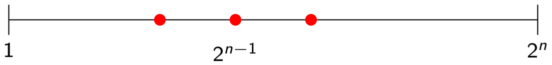

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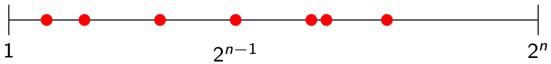

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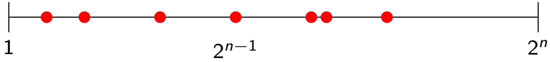

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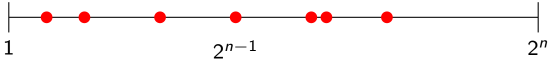

Number of balls $\propto \frac{1}{\text{position of left most ball}}$

Algorithm DE(**a**)

Choose $h : \{0,1\}^n \mapsto \{0,1\}^n$
minhash $\leftarrow 2^n$;
for $a_i \in$ **a do**
    if $h(a_i) <$ minhash **then**
        minhash $= h(a_i)$
    end if
end for
return $\frac{2^n}{\text{minhash}}$

Is there more than meets the eyes?

- From Distinct Elements to Counting
- From Counting to Distinct Elements

Hashing-based Distinct Elements

Algorithm SketchTemplate($\mathbf{a}$)

$h \leftarrow$ ChooseHashFunctions
$\mathcal{S} \leftarrow \{\}$
for $a_i \in \mathbf{a}$ **do**
    ProcessUpdate($\mathcal{S}, h, a_i$)
end for
Est $\leftarrow$ ComputeEst($\mathcal{S}$)
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Different Algorithms based on ProcessUpdate

- Minimum: Keep track of minimum $h(a_i)$
- Bucketing

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From Distinct Elements to Counting: A Two Step Recipe

$\mathbf{a}_u$: set of all distinct elements of the stream $\mathbf{a}$.

Key Idea The formula φ can viewed as symbolic representation of some set $\mathbf{a}_u$ such that $\mathsf{Sol}(\varphi) = \mathbf{a}_u$.

Step 1 Capture the relationship $\mathcal{P}(\mathcal{S}, h, \mathbf{a}_u)$ between the sketch $\mathcal{S}$, h, and the set $\mathbf{a}_u$ at the end of stream.

Step 2 Given a formula φ and hash function h, design an algorithm to construct sketch $\mathcal{S}$ such that $\mathcal{P}(\mathcal{S}, h, \mathsf{Sol}(\varphi))$ holds. And now, we can estimate $|\mathsf{Sol}(\varphi)|$ from $\mathcal{S}$.

Min-based Estimation

Algorithm minDE($\mathbf{a}$)

```
Choose h : {0,1}^n ↦ {0,1}^n
minhash ← 2^n;
for a_i ∈ a do
    if minhash < h(a_i) then
        minhash = h(a_i)
    end if
end for
return 2^n / minhash
```

Application I: Min-based Counting Algorithm

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Bucketing-based Streaming Algorithm

Algorithm BucketDE(**a**)

```
Choose h : {0,1}^n ↦ {0,1}^n
ℓ ← 0; B ← ∅
for a_i ∈ a do
    if h(a_i) mod 2^ℓ = 0^ℓ then
        B.Append(a_i)
        if |B| ≥ thresh then
            ℓ++
            Filter(B, h, ℓ)
        end if
    end if
end for
return |B| × 2^ℓ
```

Bucketing-based Streaming Algorithm

Algorithm BucketDE($\mathbf{a}$)

Choose $h : \{0,1\}^n \mapsto \{0,1\}^n$
$\ell \leftarrow 0; \mathcal{B} \leftarrow \emptyset$
for $a_i \in \mathbf{a}$ **do**
    if $h(a_i) \mod 2^\ell = 0^\ell$ **then**
        $\mathcal{B}$.Append(a_i) — Elements that satisfy XOR
        if $|\mathcal{B}| \geq$ thresh **then**
            $\ell{+}{+}$
            Filter($\mathcal{B}, h, \ell$) — Add another XOR
        end if
    end if
end for
return $|\mathcal{B}| \times 2^\ell$

Application II: Bucketing-based Counting Algorithm

Step 1 Capture the relationship $\mathcal{P}(\mathcal{S}, h, \mathbf{a}_u)$ between the sketch $\mathcal{S}$, hash function h and set $\mathbf{a}_u$ at the end of stream.

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This is ApproxMC!

From Distinct Elements to Counting: Implications

Given a formula φ and hash function h, design an algorithm to construct sketch $\mathcal{S}$ such that $\mathcal{P}(\mathcal{S}, h, \mathsf{Sol}(\varphi))$ holds.

Theorem (FPRAS)

If construction of sketch $\mathcal{S}$ is in PTIME for a class of formulas, then there is FPRAS for the corresponding class. E.g.: DNF, Union of XORs

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$p(n)$ space algorithms in streaming imply $(p(n))^2$ NP query complexity algorithms for model counting

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Theorem (Lower Bounds)

Lower bounds for Distributed Streaming translate to lower bounds for Distributed DNF counting

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- A stream can be viewed as a DNF
  - $a = a_1, a_2, a_3, \dots a_m$

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- A general scheme for structured sets
- Encompasses models such as ranges, affine spaces
- Application: Distinct Elements over Range
  - Every item $[a_i, b_i]$ can be represented using a DNF formula.
  - So just apply FPRAS for DNF

Conclusion

Summary

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Conclusion

Summary

- From Distinct Elements to Counting
- From Counting to Distinct Elements

Future Directions

- Practical scalability of newly devised counting techniques
- What's the relationship for other problems between circuits/formulas and streaming ?
    - Higher moments
    - Entropy