Efficient Enumeration Algorithms via Circuits

Antoine Amarilli

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Télécom Paris

General motivation (for database people)

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Structure of the talk:

- Preliminaries and problem statement

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Structure of the talk:

- Preliminaries and problem statement
- Efficient enumeration for **d-DNNF set circuits**
- Applications: Using enumeration on circuits for **query evaluation**

Dramatis Personae


Antoine Amarilli


Pierre Bourhis


Florent Capelli


Louis Jachiet


Stefan Mengel


Mikaël Monet


Martín Muñoz


Matthias Niewerth


Cristian Riveros

- Amarilli, A., Bourhis, P., Jachiet, L., and Mengel, S.
  A Circuit-Based Approach to Efficient Enumeration. ICALP 2017.
- Amarilli, A., Bourhis, P., and Mengel, S.
  Enumeration on Trees under Relabelings. ICDT 2018.
- Amarilli, A., Bourhis, P., Mengel, S., and Niewerth, M.
  Constant-Delay Enumeration for Nondeterministic Document Spanners. ICDT 2019.
- Amarilli, A., Bourhis, P., Mengel, S., and Niewerth, M.
  Enumeration on Trees with Tractable Combined Complexity and Efficient Updates. PODS 2019.
- Amarilli, A., Bourhis, P., Mengel, S., and Niewerth, M.
  Constant-Delay Enumeration for Nondeterministic Document Spanners. TODS 2020.
- Amarilli, A., Jachiet, L., Muñoz, M., and Riveros, C.
  Efficient Enumeration for Annotated Grammars. PODS 2022.
- Amarilli, A., Bourhis, P., Capelli, F., Monet, M.
  Ranked Enumeration for MSO on Trees via Knowledge Compilation. Under review.

Preliminaries

Enumeration algorithms (see Nofar's talks, September workshop)

Input

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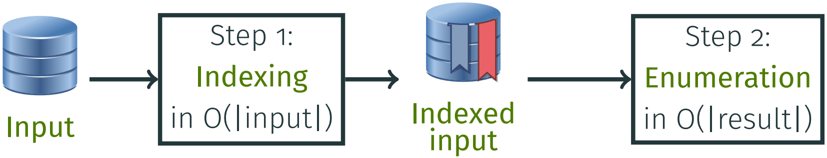

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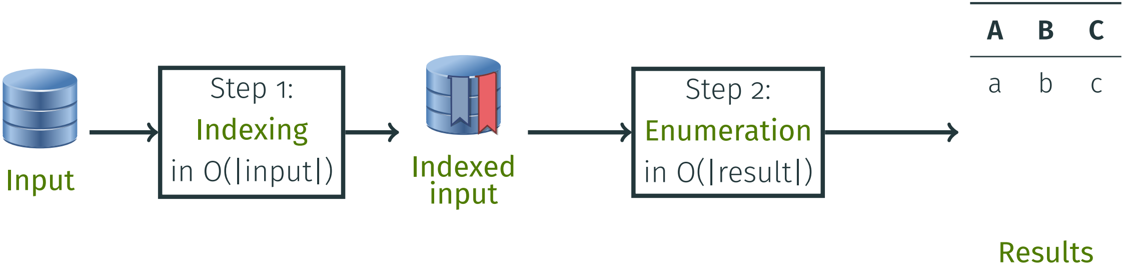

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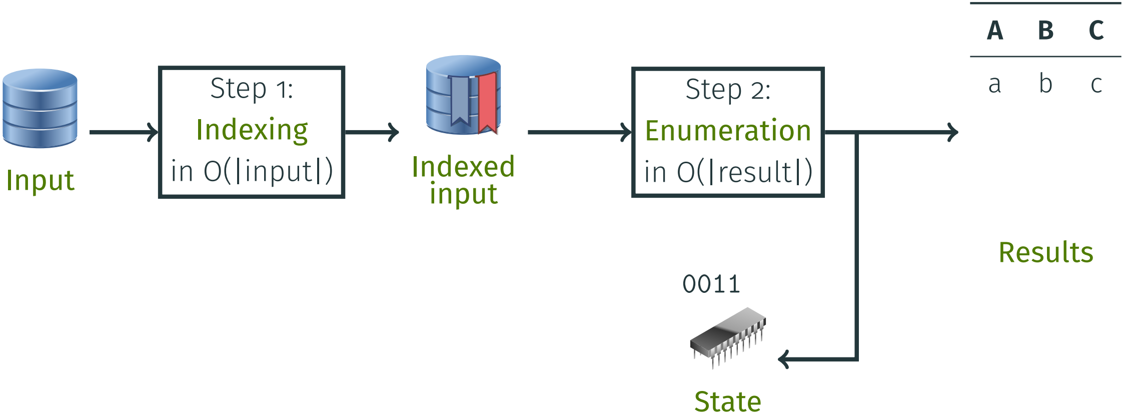

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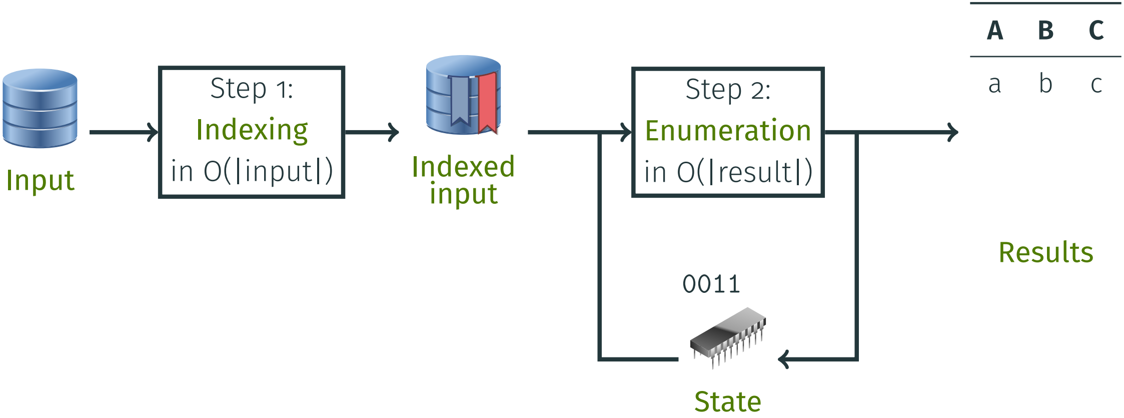

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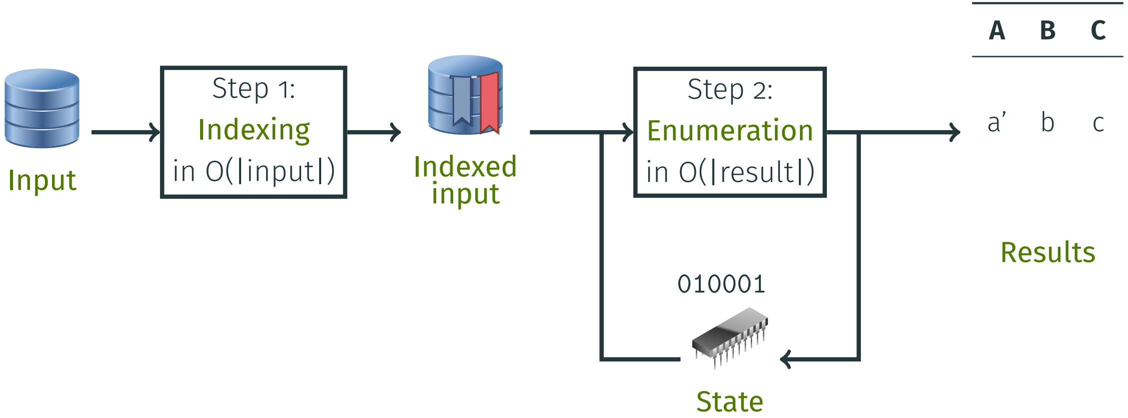

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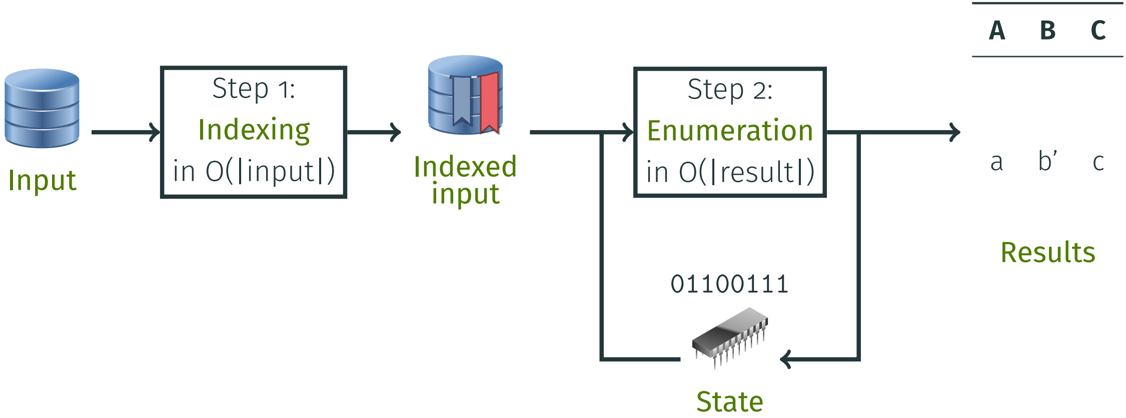

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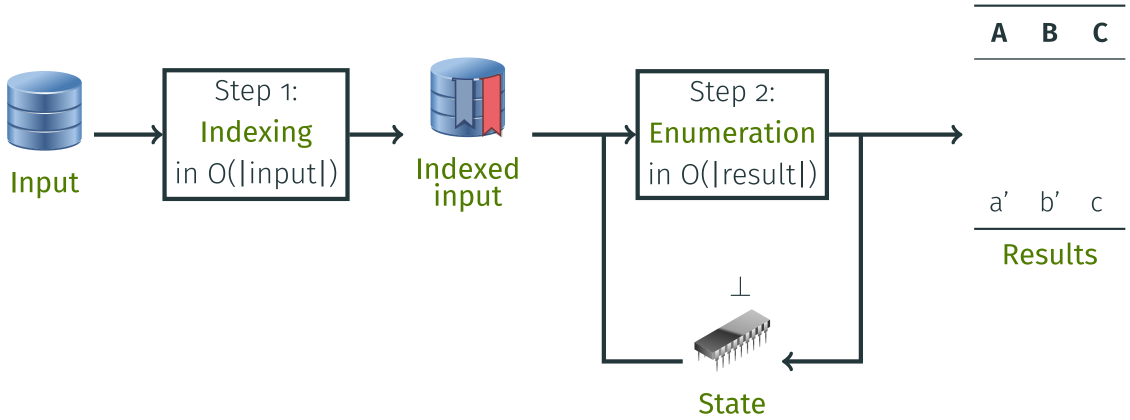
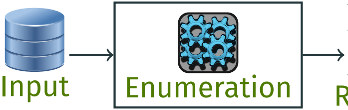


WITHOUT knowledge compilation:



A	B	C
a	b	c
a	b'	c

Results

Knowledge compilation (see Guy and YooJung's talks, Boot camp)

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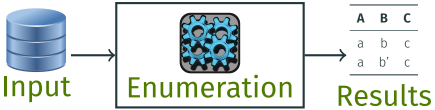

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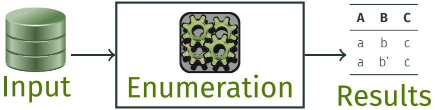

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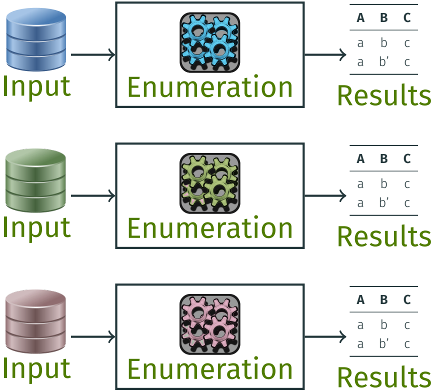

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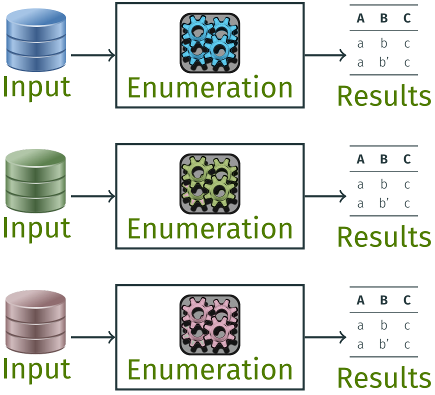

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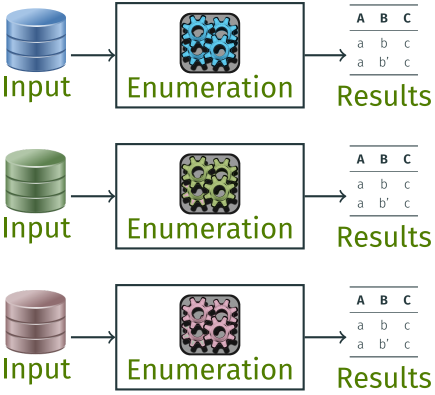

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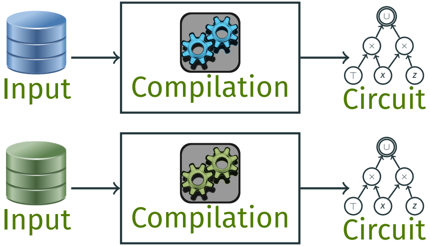

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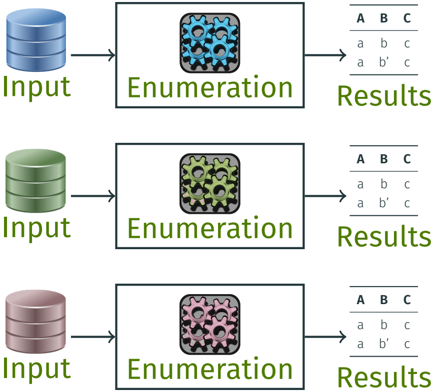

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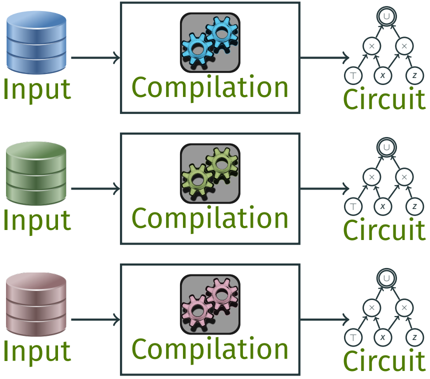

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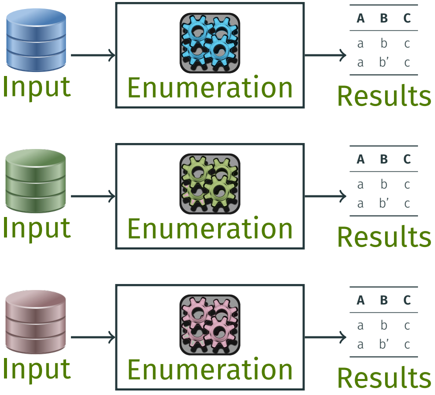

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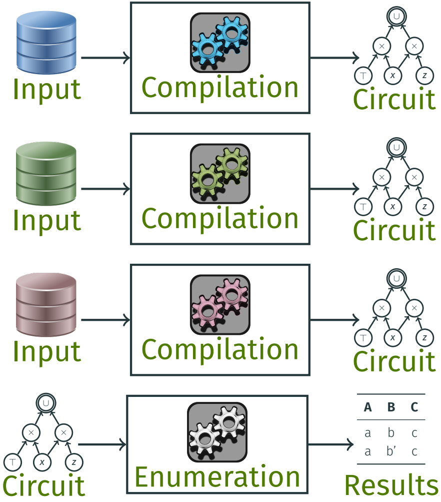
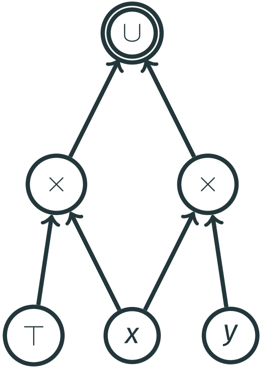
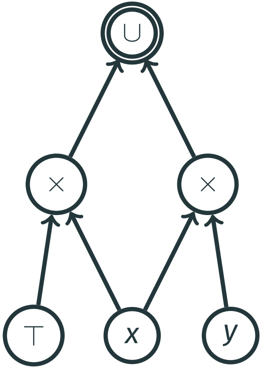


Set circuits



- Directed acyclic graph of **gates**

Set circuits



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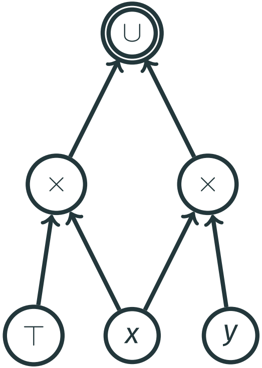

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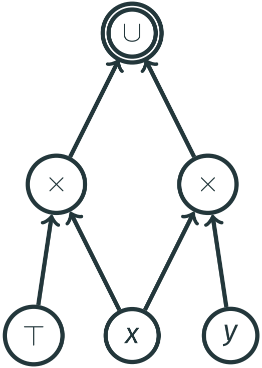

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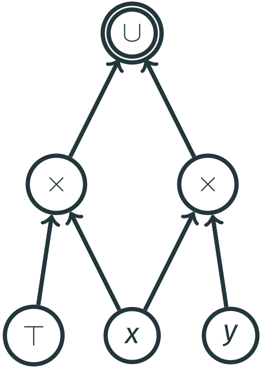

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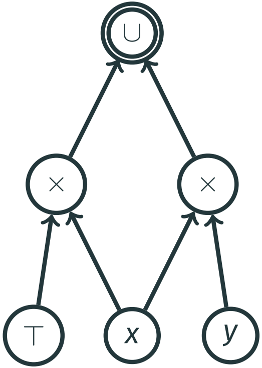

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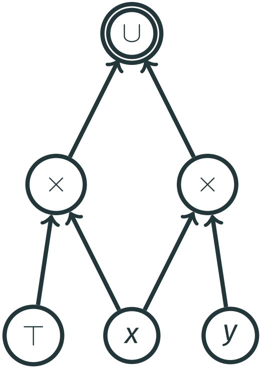
Factorized database fans may find these eerily familiar

Semantics of set circuits



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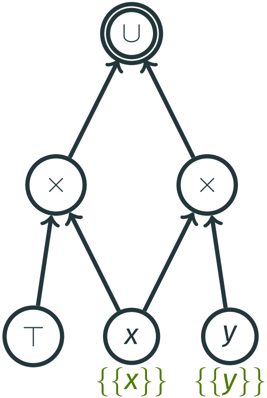

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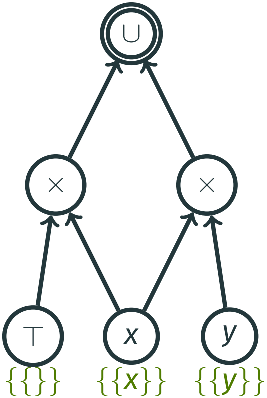

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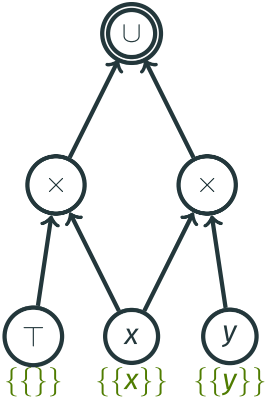

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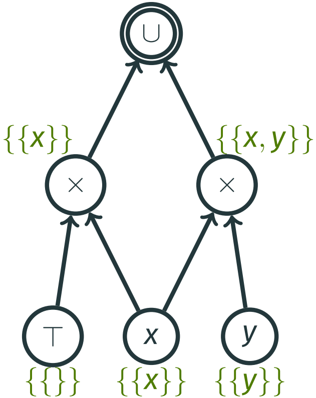

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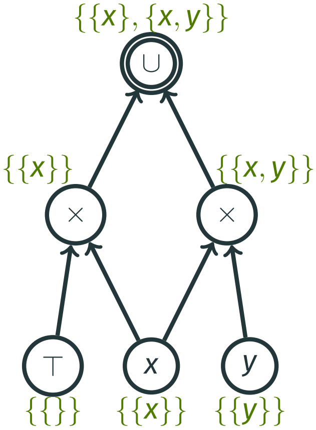

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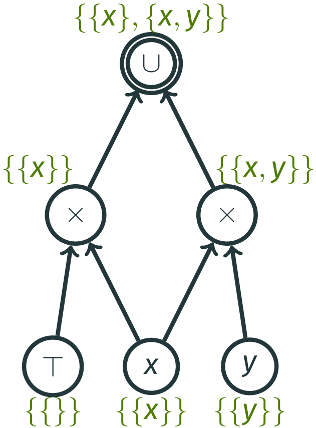

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Arithmetic circuit aficionados may see a connection

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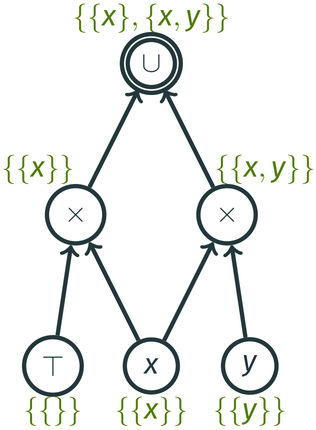

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Semiring supporters may have recognized Why$[X]$

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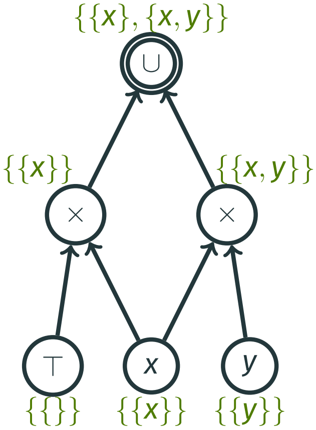

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Task: Enumerate the assignments of the set $S(g)$ captured by a gate g

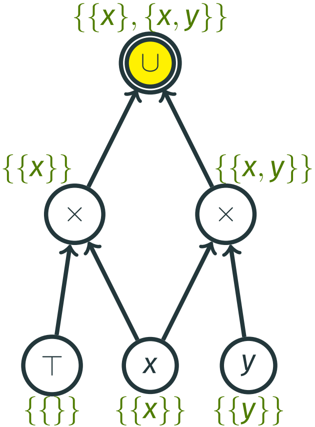
$\rightarrow$E.g., for $S(g) = \{\{x\},\{x,y\}\}$, enumerate $\{x\}$ and then $\{x,y\}$

Circuit restrictions

d-DNNF set circuit:

- ∪ are all **deterministic**:

The inputs are **disjoint**
(= no assignment is captured by two inputs)



Circuit restrictions

d-DNNF set circuit:

- $\cup$ are all **deterministic**:

The inputs are **disjoint**
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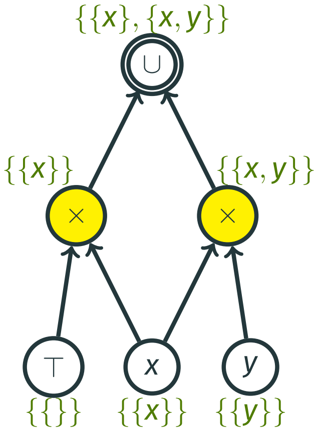
- $\times$ are all **decomposable**:

The inputs are **independent**
(= no variable x has a path to two different inputs)



Main results

Theorem (A., Bourhis, Jachiet, Mengel, ICALP'17)

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  - From a **d-DNNF**, in **quadratic time** (smoothing)

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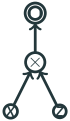
    - From a **d-DNNF**, in **quadratic time** (smoothing)
    - From a **d-SDNNF**, in **linear time** when allowing special gates (implicit smoothing)

Proof techniques

Proof overview

Preprocessing phase:



d-DNNF
set circuit

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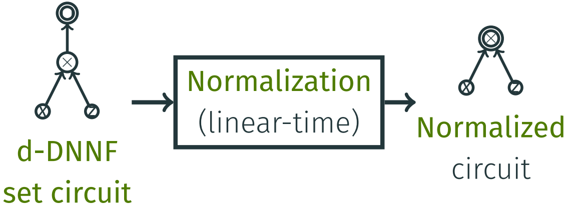

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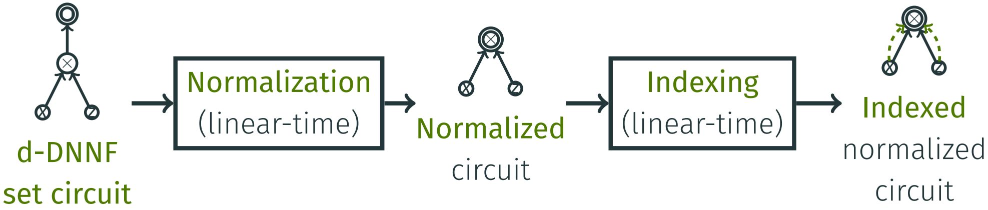

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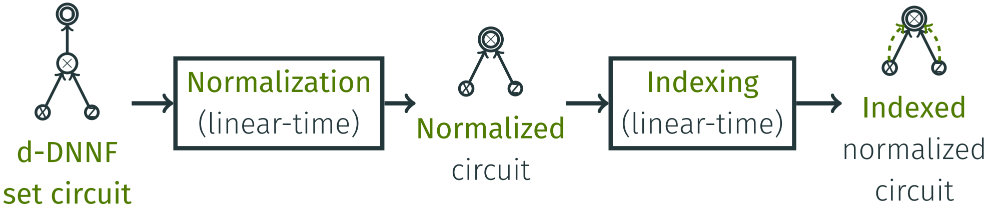
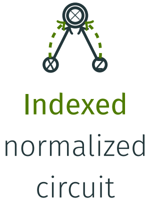


Enumeration phase:



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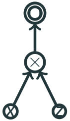

d-DNNF set circuit → **Normalization** (linear-time) →

Normalized circuit → **Indexing** (linear-time) →

Indexed normalized circuit

Enumeration phase:

Indexed normalized circuit → **Enumeration** (linear delay in each result) →

A	B	C
a	b	c
a	b’	c

Results

Enumerating captured assignments of d-DNNF set circuits

Task: Enumerate the assignments of the set $S(g)$ captured by a gate g

→ E.g., for $S(g) = \{\{x\}, \{x, y\}\}$, enumerate $\{x\}$ and then $\{x, y\}$

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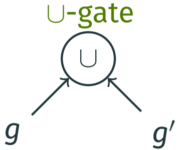

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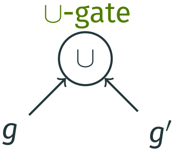

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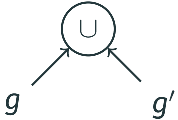
$\cup$-gate



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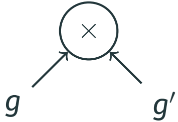
$\times$-gate



Lexicographic product: enumerate $S(g)$ and for each result t enumerate $S(g')$ and concatenate t with each result

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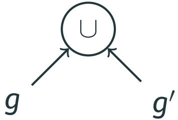

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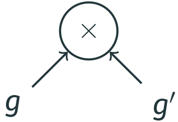

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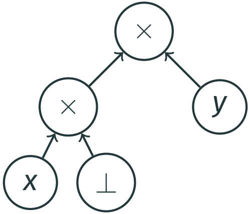
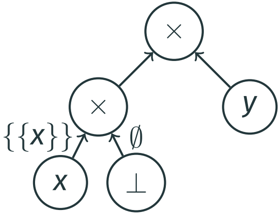
Decomposability: no duplicates

Normalization: handling ∅



Normalization: handling $\emptyset$



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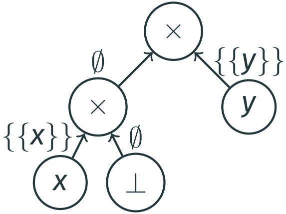

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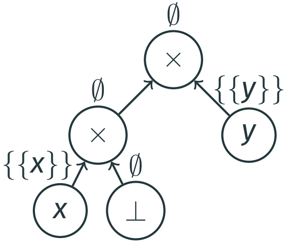

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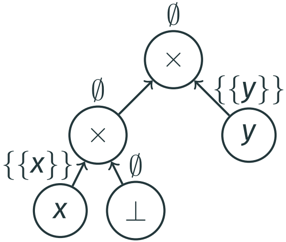

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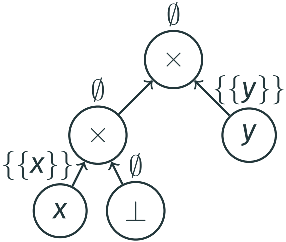

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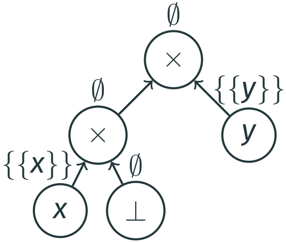

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  - compute **bottom-up** if $S(g) = \emptyset$
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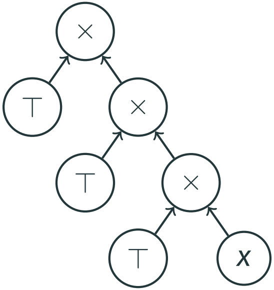

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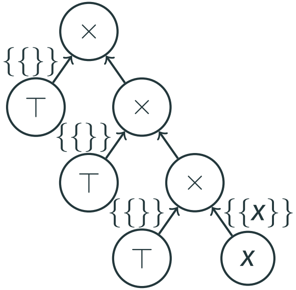

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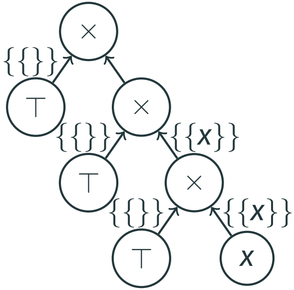

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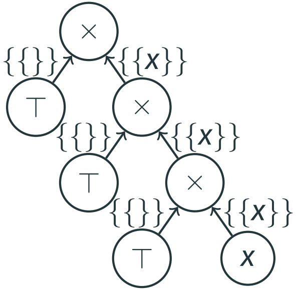

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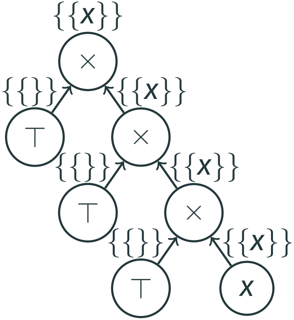

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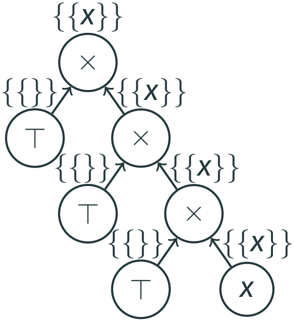

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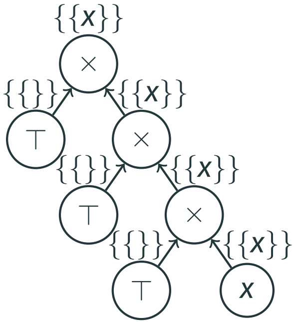

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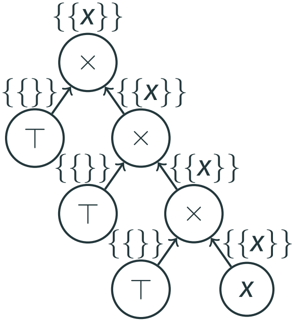

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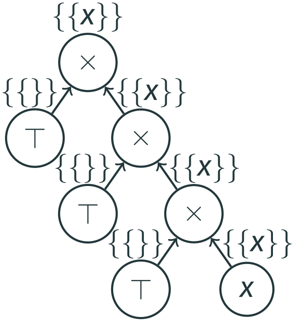

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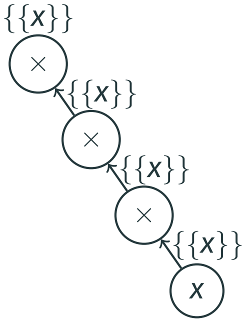

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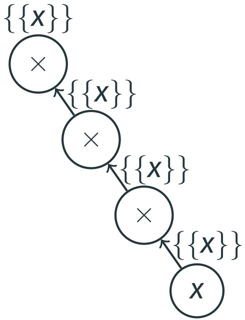

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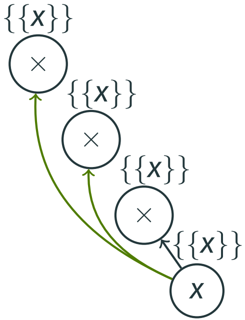

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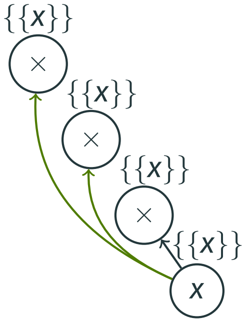

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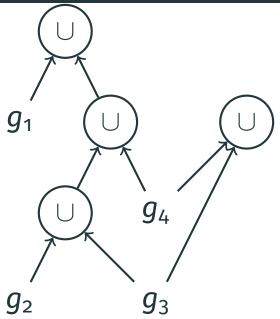
$\rightarrow$ Now, when traversing a **$\times$-gate** we make progress: **non-trivial split** of each set

Indexing: handling ∪-hierarchies



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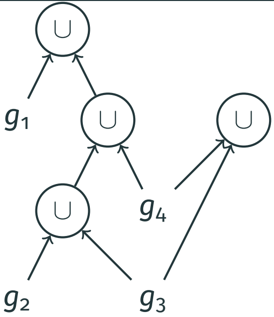

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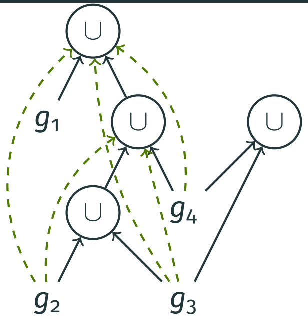

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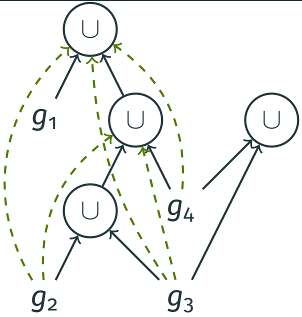

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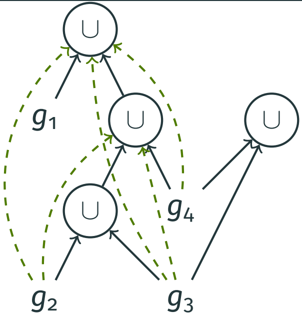

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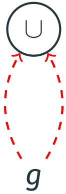

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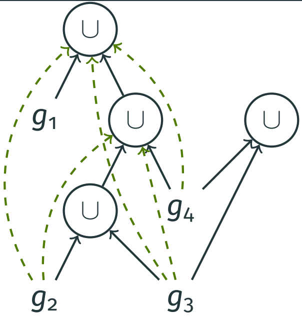

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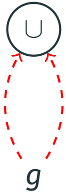
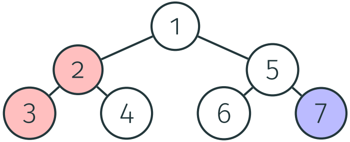
- **Custom** constant-delay reachability index for multitrees



Applications

Application 1: MSO query evaluation on trees

Data: a **tree** T where nodes have a color from an alphabet



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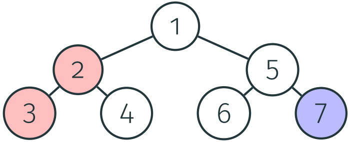

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Query Q in monadic second-order logic (MSO)

- $P_{\bigcirc}(x)$ means "x is blue"
- $x \rightarrow y$ means "x is the parent of y"

"Find the pairs of a pink node and a blue node?"

$Q(x, y) := P_{\bigcirc}(x) \wedge P_{\bigcirc}(y)$

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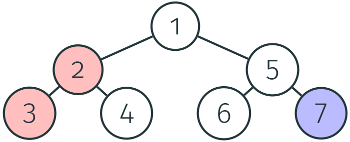

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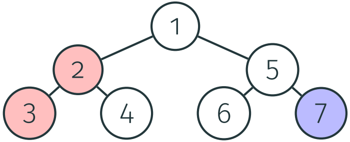

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results: $(2, 7), (3, 7)$

Data complexity: Measure efficiency as a function of T (the query Q is **fixed**)

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Theorem [Bagan, 2006, Kazana and Segoufin, 2013]

We can enumerate the answers of MSO queries on trees with linear-time preprocessing and constant delay.

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- Same result for leaf **insertion/deletion** (A., Bourhis, Mengel, Niewerth, PODS'19) up to **fixing a buggy result** [Niewerth, 2018]

Application 2: Enumerating matches of nondeterministic document spanners

Data: a **text** T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.  French national.  Appearance as
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test@example.com More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
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Application 2: Enumerating matches of nondeterministic document spanners

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$$[186, 200\rangle, \quad [483, 500\rangle, \quad \dots$$

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Goal:

- be **very efficient** in T (constant-delay)
- be **reasonably efficient** in P (polynomial-time)

Application 2: Results

Theorem (A., Bourhis, Mengel, Niewerth, ICDT'19)

*We can enumerate all matches of an input **nondeterministic automaton with captures** on an input **text** with*

- *Preprocessing **linear** in the text and **polynomial** in the automaton*
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→ Generalizes to **trees** with polynomial dependency in the **tree automaton**

Application 3: Enumerating matches of annotated grammars

Data: a text *T*, e.g., source code

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long elt, prev, elt2, prev2=-1;
int ret = fscanf(fi, "%ld%ld", &elt, &prev);
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*Given an **unambiguous annotation grammar** $\mathcal{G}$ and input text w, we can enumerate the **matches** with preprocessing $O(|\mathcal{G}| \times |w|^3)$ and delay **linear in each assignment***

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- Improves on an earlier quintic preprocessing result [Peterfreund, 2021]
- Quadratic and linear preprocessing for subclasses (rigid grammars, deterministic pushdown annotators)

Other applications

- Using enumerable compact sets, a fully-persistent version of enumerable d-DNNFs:
    - For visibly pushdown transducers on nested documents in a streaming setting [Muñoz and Riveros, 2022]
    - For annotated automata on SLP-compressed documents, with updates [Muñoz and Riveros, 2023]

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- Query evaluation beyond MSO and variants on words and trees:
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- Can also be used to enumerate **homomorphisms between structures** [Berkholz and Vinall-Smeeth, 2023]

What about ranked enumeration?

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- For MSO queries, ranked enumeration is possible with logarithmic delay:
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  - (Very) high-level idea: use one **priority queue** for each gate
- For **CQs**, results for **ranked access**: [Tziavelis et al., 2022], [Deep et al., 2022], [Carmeli et al., 2023]
  - Also: see Florent's talk

Conclusion

Summary and conclusion

- We can **enumerate** the captured assignments of d-DNNF set circuits
  - → with preprocessing **linear** in the d-DNNF
  - → in delay **linear** in each assignment
  - → in **constant** delay for constant Hamming weight

→ Applies to **MSO enumeration** on **words** and **trees**

→ Applies to enumerate of the matches of **annotated context-free grammars** (with more expensive preprocessing)

→ Can be used for **other applications**

→ In particular: **incremental maintenance** under updates, **ranked enumeration**, etc.

Questions for future work

- What about **negation gates**?
- What can we do without **determinism**? (enumeration for DNNF?)
- Connect results on **updates** to finer bounds on **incremental maintenance** (A., Jachiet, Paperman, ICALP'21)
- Enumerate satisfying assignments via **edits on previous results** (A., Monet, STACS'23) to achieve **constant delay** even on **linear-sized assignments**
- For MSO queries: understand better the connection between **automata classes** and **circuit classes** (e.g., alternating automata, two-way automata...)
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Thanks for your attention!

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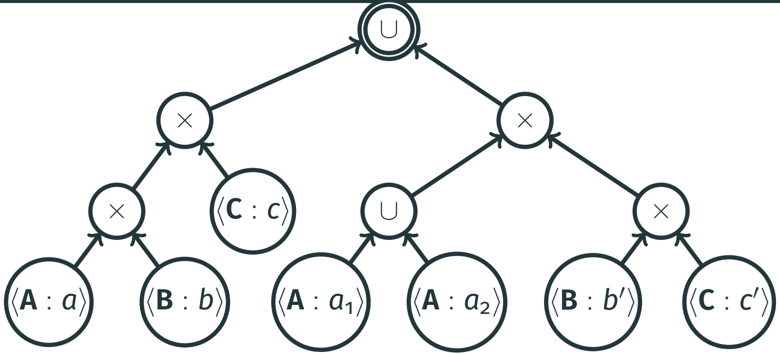
Set circuits vs factorized representations

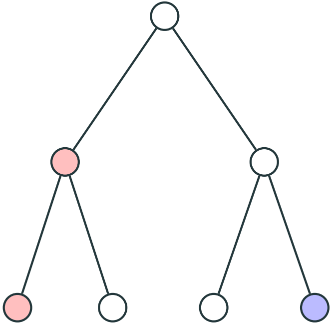
A	B	C
a	b	c
a_1	b'	c'
a_2	b'	c'



- Set circuits can be seen as **factorized representations**
  - → Not necessarily **well-typed**, height and/or assignment size may be **non-constant**
- **Determinism**: unions are disjoint
- **Decomposability**: no duplicate attribute names in products
- **Structuredness**: always the same decomposition of the attributes

Tree automata

Tree alphabet: 



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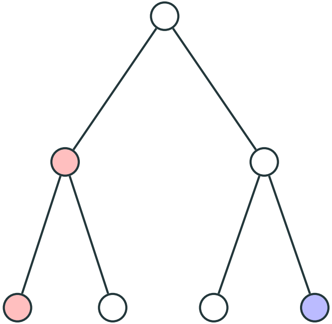

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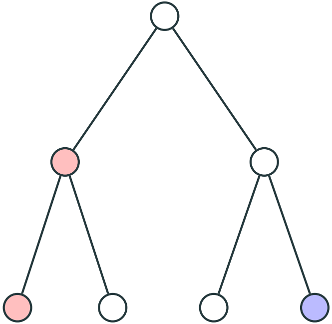

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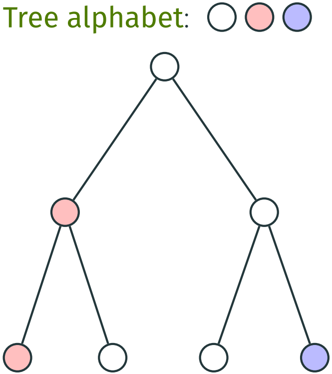

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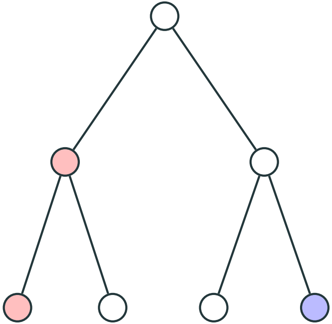

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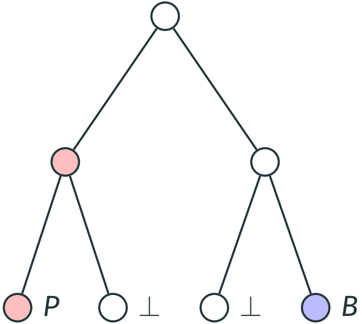

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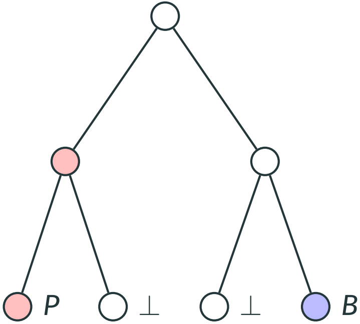

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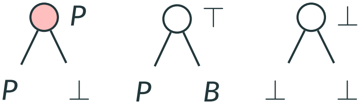

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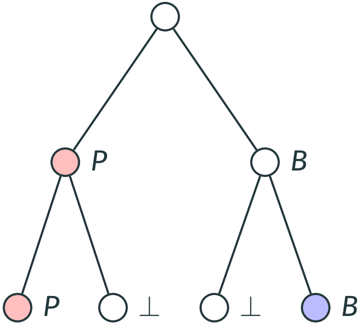

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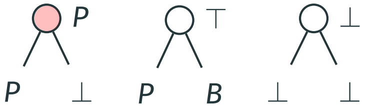

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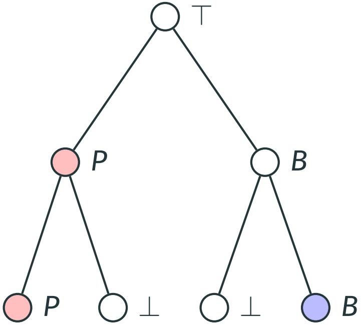

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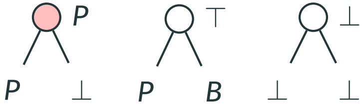
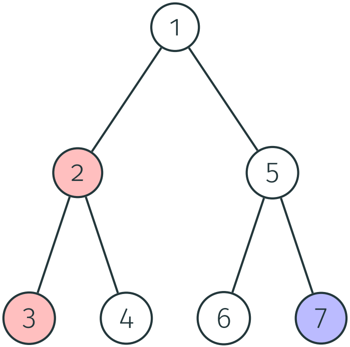


Uncertain trees

Now: Boolean query on a tree where the color of nodes is uncertain



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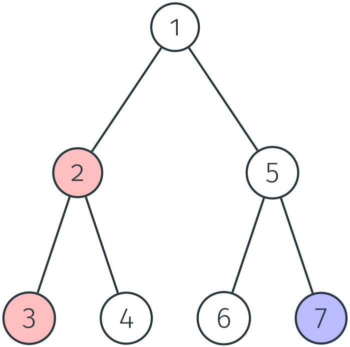

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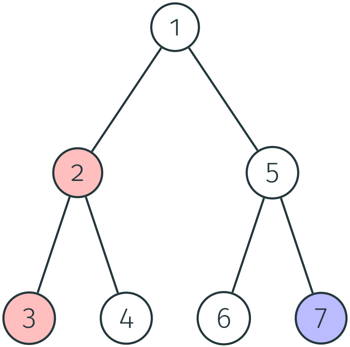

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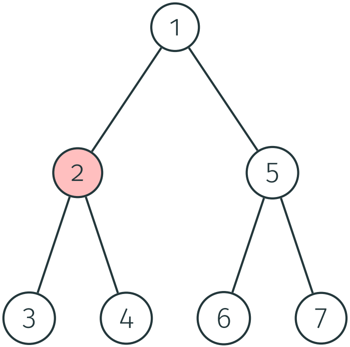

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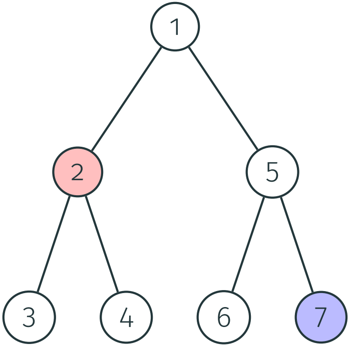

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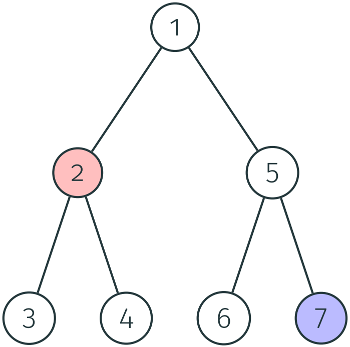

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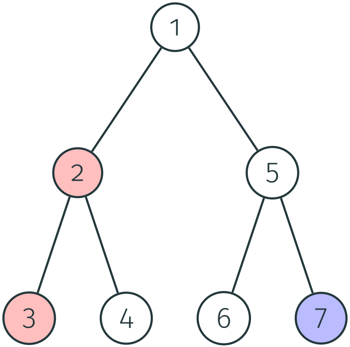

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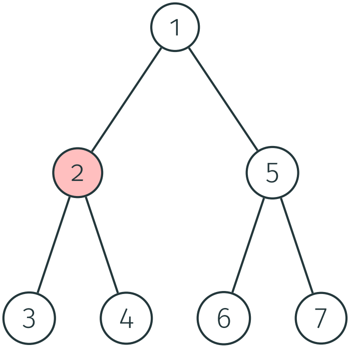

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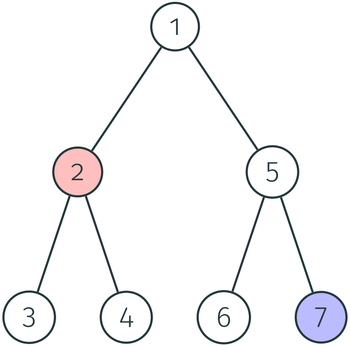

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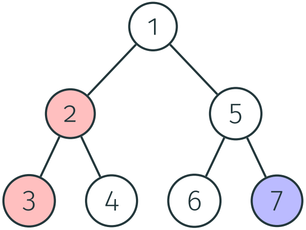

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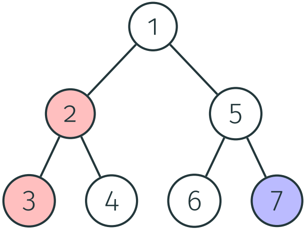

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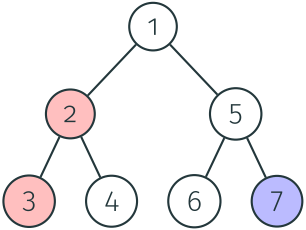

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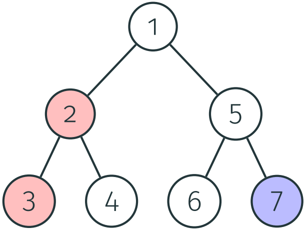

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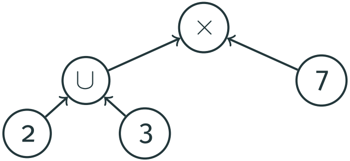

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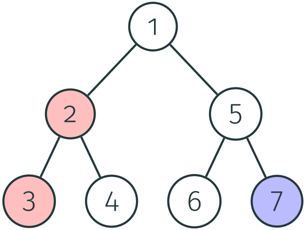

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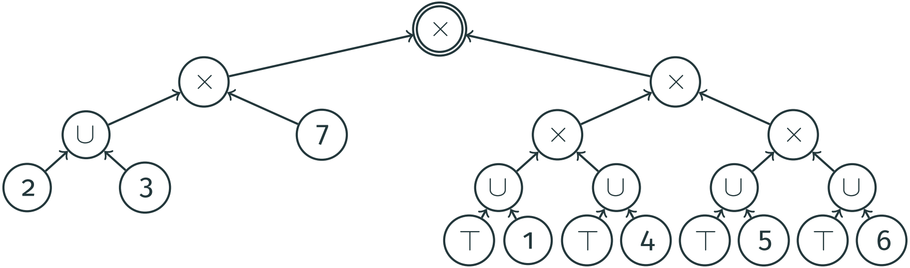

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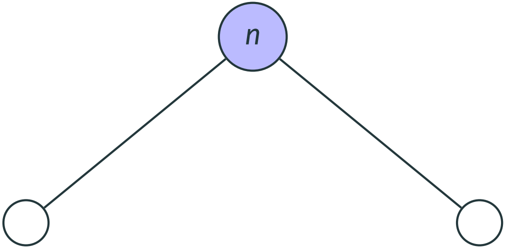

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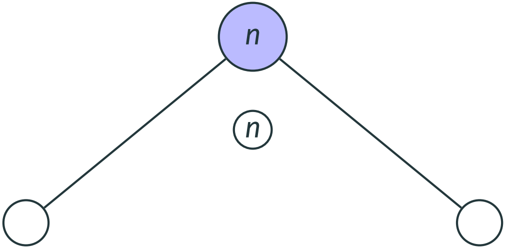

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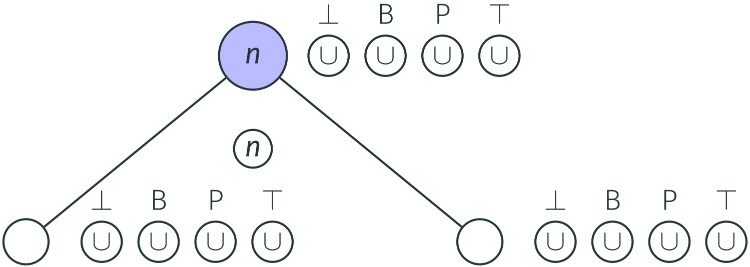

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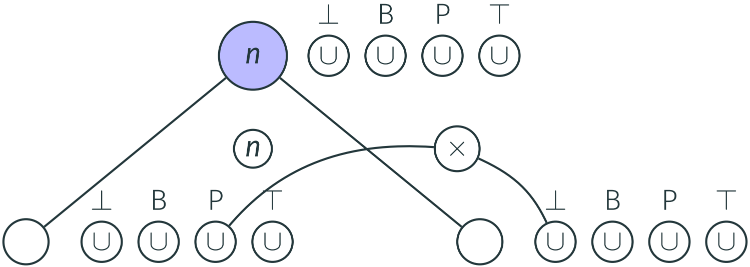

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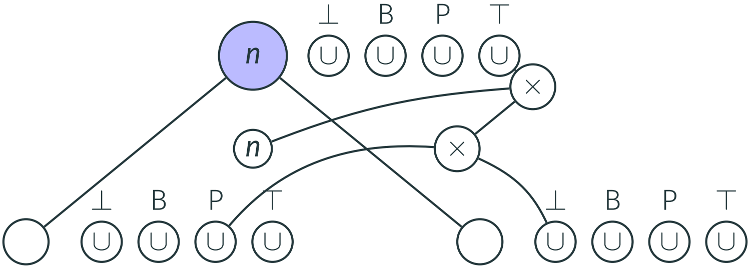

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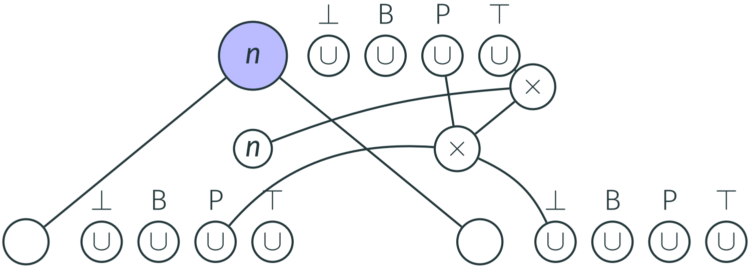

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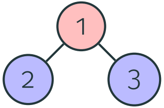

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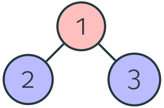

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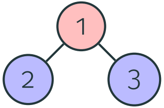

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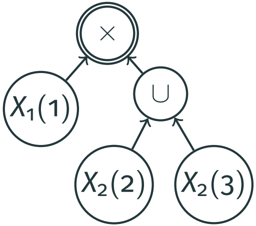

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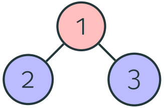

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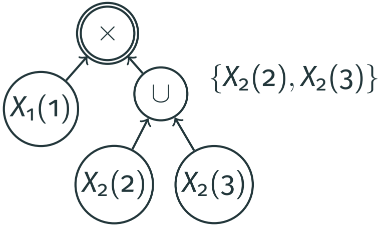

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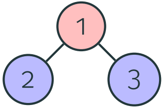

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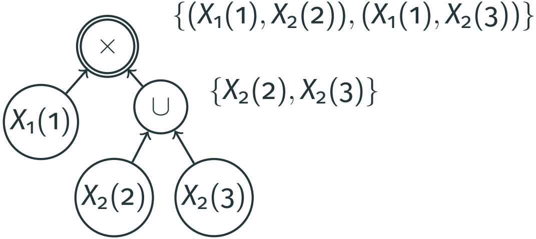

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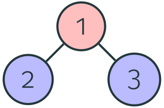

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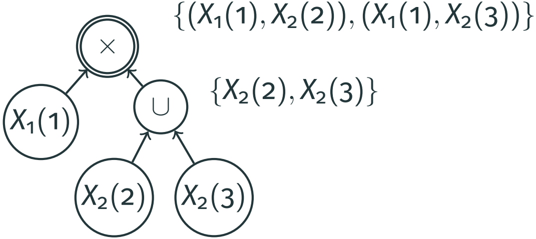

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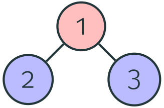

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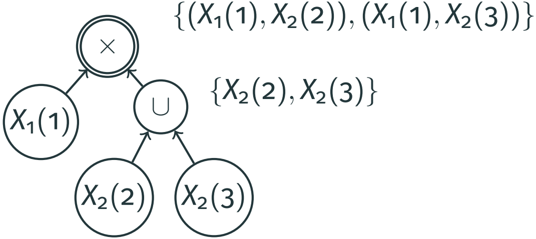

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Semi-open question: what about memory usage?