Approximating k-Edge Connected Spanning Subgraph via a Fast LP Solver

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Joint work with

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kECSS problem

mincut $\geq k$

- A graph is *k-edge-connected* if it remains connected after removing k-1 edges
- **Input**: An undirected graph $G = (V, E)$ and a cost function c on edges $(n := |V|, m := |E|)$
  Output: A min-cost k-edge-connected spanning subgraph $H = (V, E'), E' \subseteq E$
  - **Example**: k = 1 is easy (why?)

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  - APX-hard for k > 1 even on bounded degree graphs and binary cost function [Pritchard'10]

  - Best known approximation is 2 [Khuller and Vishkin, STOC'92]

  - A special case of survivable network design problem (SNDP) [Jain'01]

History

- m-edge n-vertex graph G
- ignore polylog factor

	Approx.	Time	Remark
Frederickson Jaja'81	3	n^2	for k = 2 (2-ECSS)
Khuller Vishkin'94	2	nmk	
Gabow Goemans Williamson'98	2k-1	n^2	
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This work	$2+\epsilon$	$O(m \cdot \epsilon^{-2})$	**Only output the estimate**
This work	$2+\epsilon$	$O(\epsilon^{-2} \cdot (m + n^{1.5}k^2))$	

Fast LP Solvers

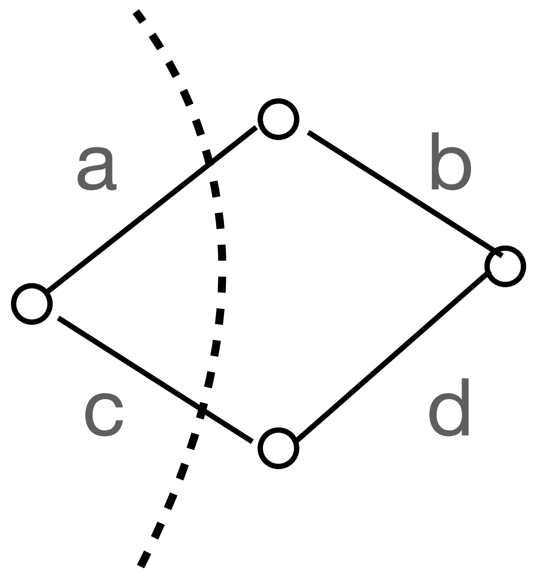
kECSS LP: $\min \sum_{e \in E} c_e x_e$

For all cut C, $\sum_{x_e \in C} x_e \geq k$

$x \in [0,1]^E$

Set of edges

C = {a,b}



How fast can we solve the kECSS LP?

- $\tilde{O}(\frac{mn}{\varepsilon^2})$ time [Fleischer'04]

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Theorem: (This work)

kECSS LP can be solved in $\tilde{O}(\frac{m}{\varepsilon^2})$ time

Extend the techniques from [CQ, FOCS'17] to solve the kECSS LP

Applications of the kECSS LP solver

- We can estimate the optimal integral solution within $(2+\epsilon)$ factor

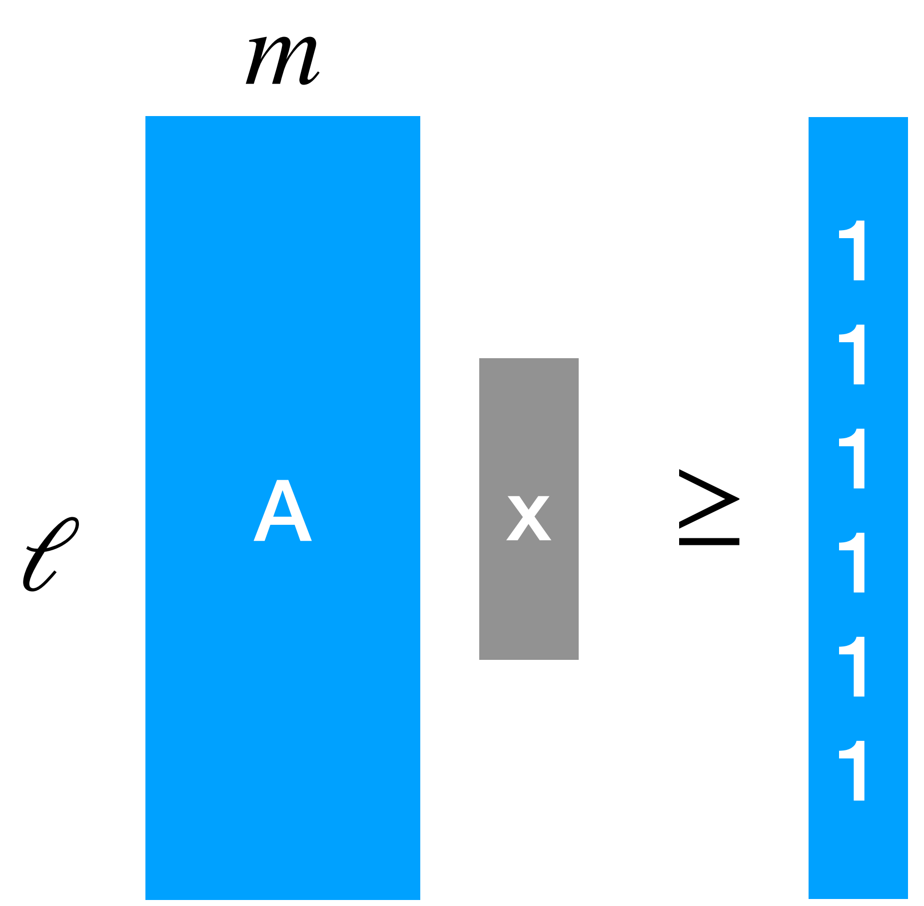
  (since the integrality gap of the kECSS LP is 2 [KV'94])

- $(2+\epsilon)$ Integral solution to kECSS in time $\tilde{O}(\frac{m}{\varepsilon^2} + n^{1.5}k^2\varepsilon^{-2})$
  - **Approach:**
    (1) Solve kECSS LP $m\epsilon^{-2}$ $m' = \tilde{O}(k^2 n \epsilon^{-2})$
    (2) Sparsify the LP solution by using the graph compression theorem from [Benzcur Karger'15]
    (3) run [Khuller Vishkin'94] algorithm on the support of the sparsified graph
    $\tilde{O}(m'\sqrt{n}) = \tilde{O}(k^2 n^{1.5} \epsilon^{-2})$

Preliminaries: MWU framework

Covering LP:

$$\min c^T x$$
$$Ax \geq \vec{1}$$
$$x \geq 0$$



$A_{ij} \geq 0$

$c_i \geq 0$

MinRow Problem: given $w_t \in R^m_{\geq 0}$

$$\text{MinRow}(A, w_t) := \arg \min_{\text{row } i} \langle A_i, w_t \rangle$$

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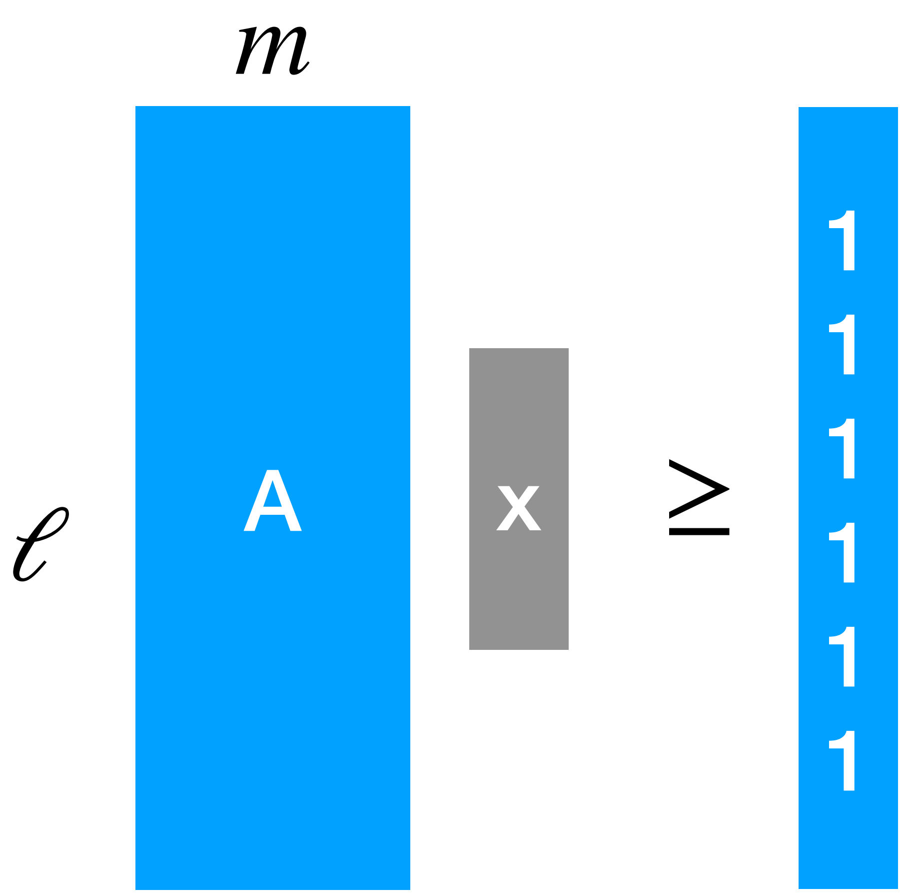

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Theorem: MWU (width-independent) [GK FOCS'98]

We can obtain $(1+\epsilon)$ -approx solution to the covering LP by solving a sequence of Minrow problems for $O(\frac{m \log m}{\epsilon^2})$ iterations

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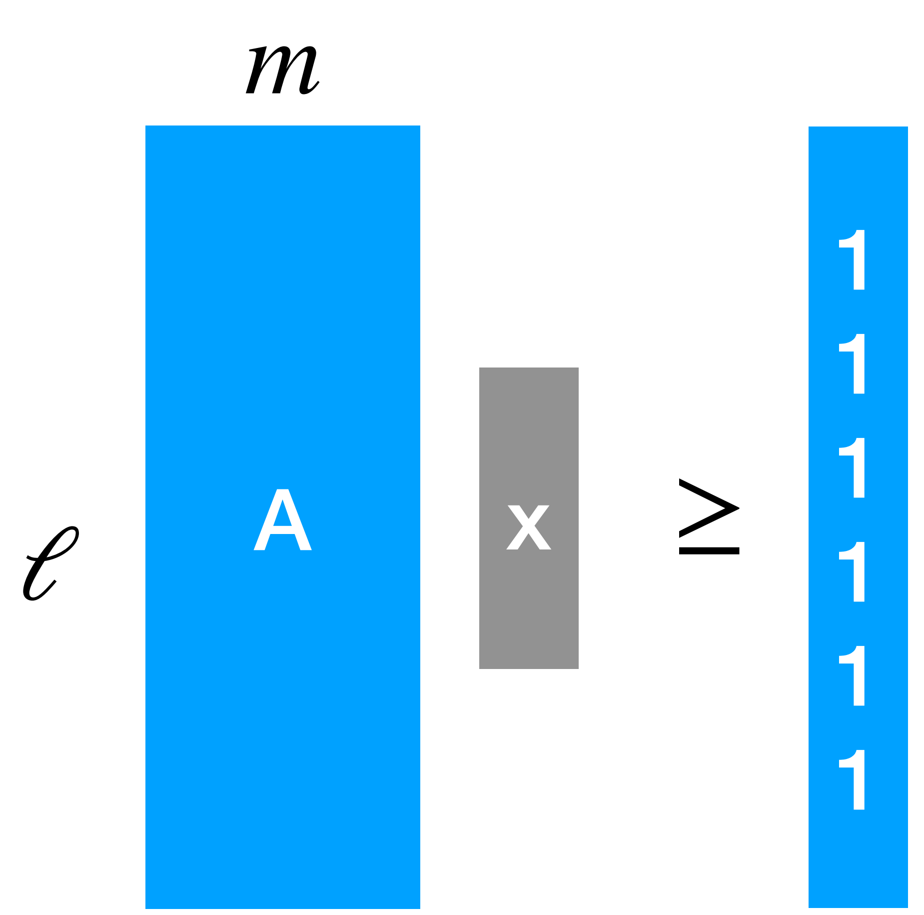

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Omri's talk on
Accelerating the Multiplicative-Weights Framework
(Workshop on Dynamic graphs and Algorithm Design)

For example,

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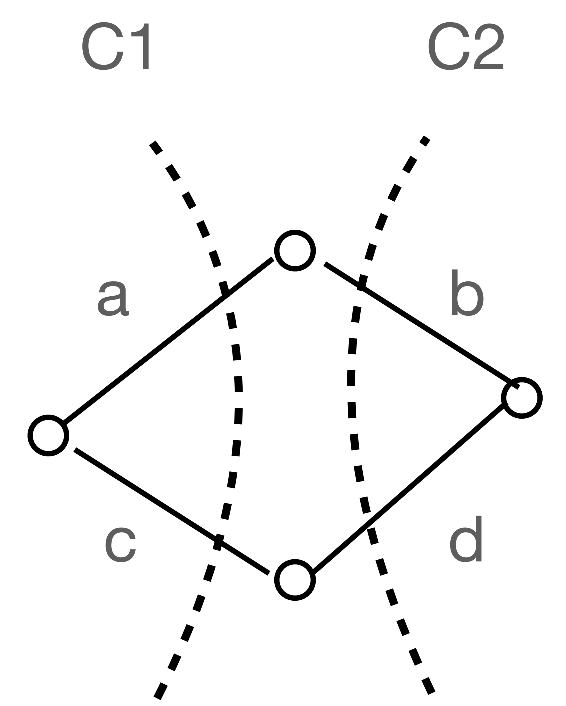
$$A = \begin{array}{c} \\ C1 \\ C2 \\ \vdots \end{array} \begin{array}{cccc} a & b & c & d \\ \hline 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ & \cdots & & \end{array}$$



$w : E \to \mathbb{R}_{\geq 0}$ MinRow$(A, w_t) := \arg \min_{\text{row } i} \langle A_i, w_t \rangle$

(Global) mincut problem

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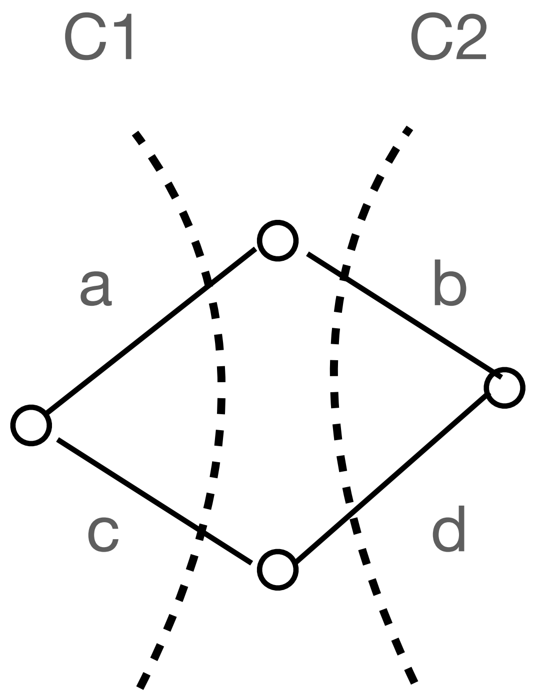

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- To apply MWU algorithm, need a pure covering LP
- To get rid of the box-constraints, add extra inequalities

  aka, knapsack cover inequality [CFLP'00]

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kECSS Pure: $\min \sum_{e \in E} c_e x_e$

For each $F \subseteq C, |F| < k,$ $\sum_{e \in C \setminus F} x_e \geq k - |F|$ **Same as** $\sum_{e \in C \setminus F} x_e / (k - |F|) \geq 1$

free-edge set

$x \geq 0$

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MinRow(A^{kc}, w_t) = ?

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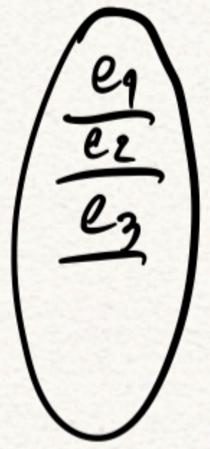
C_1



$k=3,$

A = cut-incidence matrix

	e_1	e_2	e_3	e_4	e_5	...
C_1	1	1	1	0	0	

⇒

A^{kc}

	e_1	e_2	e_3	e_4	e_5
$(C_1, \emptyset)$	$1/k$	$1/k$	$1/k$	0	0
$(C_1, \{e_1\})$	0	$1/k-1$	$1/k-1$	0	0
$(C_1, \{e_2\})$	$1/k-1$	0	$1/k-1$	0	0
$(C_1, \{e_3\})$	$1/k-1$	$1/k-1$	0	0	0
$(C_1, \{e_1, e_2\})$	0	0	$1/k-2$	0	0

⋮

$$\text{MinRow}(A^{kc}, w) = \min_{(C,F)} A^{kc}_{(C,F)} \cdot w = \min_{\substack{C, F \subseteq C \\ |F| < k}} \frac{w(C \setminus F)}{k - |F|}.$$

- where $w(S) := \sum_{e \in S} w(e)$

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kECSS Pure:

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where A' is a (cut,free-edge)-matrix of G obtained by adding above inequalities

- **Fact**: These two LPs are equivalent
- MinRow(A',w) = ?

The Normalized Free Cut

$$\text{MinRow}(A^{kc}, w) := \min_{\text{row}(C,F)} < A^{kc}_{(C,F)}, w > = \min_{C, F\subseteq C, |F|<k} \frac{w(C\setminus F)}{k - |F|}$$

Mincut

$$\text{MinRow}(A, w) := \min_{C} w(C)$$

- **Upshot**: removing the box constraint makes the MinRow problem more complex

The Normalized Mincut Problem

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"Free" up to k-1 edges

The **more** free, the **less** normalizing factor

Let $\lambda_i = \min_{C, F \subseteq C, |F| = i} w(C \backslash F) =$ Mincut with i edges being free

$$\text{MinRow}(A^{kc}, w) := \min\{\frac{\lambda_0}{k}, \frac{\lambda_1}{k-1}, \ldots, \frac{\lambda_{k-1}}{1}\}$$

Brute-force?

"Free" up to k-1 edges

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Fast Algorithm ?

Fact: Computing $\lambda_i =$ connectivity interdiction problem in time $O(mn^4)$ [Zenklusen14]

Thus: $\text{MinRow}(A^{kc}, w)$ in $O(kmn^4) = O(m^2 n^4)$ Time Fastest known (Based on cut enumeration)

Furthermore, we need dynamic algorithm for maintaining MinRow problems

The Normalized Mincut Problem

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Main Technical Results

Theorem: (Static MinRow)

$\text{MinRow}(A^{kc}, w)$ can be approximated in $\tilde{O}(m\epsilon^{-1})$ time

Reduction to the mincut problem

Theorem: (Dynamic MinRow)

$\text{MinRow}(A^{kc}, w)$ can be dynamically maintained under the weight w increases by MWU in $\tilde{O}(m\epsilon^{-2})$ total time

Reduction to the dynamic mincut algorithm of [Chekuri, Quanrad, FOCS'17]

Static Reduction

- **Input**: a weighted graph G = (V,E,w)
- **Output**: $(1+\epsilon)$-normalized min-cut in nearly linear time
- Denote $\text{OPT} = \textsc{MinRow}(A^{kc}, w) = \min_{C, F \subseteq C, |F| < k} \frac{w(C \setminus F)}{k - |F|}$
- Let $\rho := (1+\epsilon)\text{OPT}$
- Truncated Weight: $w_\rho(e) = \min\{w(e), \rho\}$

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- **Lemma 1**: Let C be the minimum cut wrt. w_ρ . Then, there exists a set of free edges $F \subseteq C, |F| < k$ such that $\frac{w(C \setminus F)}{k - |F|} \leq (1+\epsilon)\text{OPT}$
- **Theorem 2** [Karger, STOC'96]: minimum cut can be solved in nearly-linear time

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Algorithm:

1. Let C* be the minimum cut wrt. w_ρ
2. Find F* of size less than k that minimizes $\frac{w(C^* \setminus F^*)}{k - |F^*|}$
3. **Return** (C*,F*)

- **Lemma 1**: Let C* be the minimum cut wrt. w_ρ . Then, there exists a set of free edges $F \subseteq C^*, |F| < k$ such that $\frac{w(C^* \setminus F)}{k - |F|} \leq (1+\epsilon)\text{OPT}$

- **Proof:** (for any cut C)

  - **Claim 1:** If $w_\rho(C) < k(1+\epsilon)\text{OPT}$, then, there exists a set $F \subseteq C, |F| < k$ such that $\frac{w(C \setminus F)}{k - |F|} \leq (1+\epsilon)\text{OPT}$

  - **Claim 2:** $w_\rho(C^*) < k(1+\epsilon)\text{OPT}$

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Therefore, $w_\rho(D) < (1+\epsilon)k\text{OPT}$

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    $$\leq (k - |F|)(1+\epsilon)\text{OPT}$$

    Therefore, $\frac{w(C \setminus F)}{k - |F|} \leq (1+\epsilon)\text{OPT}$ $\quad QED$

Dynamic Reduction

Theorem: (Dynamic MinRow)

MinRow(A^{kc}, w) can be dynamically maintained under the weight w increases by MWU in $\tilde{O}(\frac{m}{\epsilon^2})$ total time

Review MWU Framework
[GK'98]

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Initialize $w : E \to \mathbb{R}_{\geq 0}$

Repeat for $O(\epsilon^{-2} \cdot m \log m)$ **iterations**

- Compute (C, F) = $(1 + \epsilon)$- approximate MinRow$(A^{kc}, w) = \min_{C, F \subseteq C, |F| < k} \frac{w(C \setminus F)}{k - |F|}$
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Assume free for this talk

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Issue 1: Need to update ρ every time
Issue 2: Given C, how to identify its best free set quickly?

To handle issue 1: batch OPT into a range of the powers of $(1+\epsilon)^i$

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Epoch-based MWU Framework
[Fleischer'00]

Initialize $w : E \to \mathbb{R}_{\geq 0}$

$\lambda \leftarrow$ MinRow(A^{kc}, w)

Repeat for $O(\epsilon^{-2} \log m)$ **epochs**

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- Compute (C, F) = with normalized weight in $[\lambda, (1+\epsilon)\lambda)$
- Update weights w and repeat **until** MinRow$(A^{kc}, w) \geq (1+\epsilon)\lambda$

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Same threshold

$\lambda \leftarrow (1+\epsilon)\lambda$
start a new epoch

Review MWU Framework

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Theorem: (Dynamic MinRow) During an epoch, MinRow(A^{kc}, w) can be dynamically maintained under the weight w increases by MWU in $\tilde{O}(m)$ total time

For simplicity, assume #of edges is bounded $|C| = \tilde{O}(1)$

(we use the data structure from **[CQ'17] to** handle the **general** case)

Epoch-based MWU Framework

Repeat for $O(\epsilon^{-2} \log m)$ **epochs**

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Issue 2: Given C, how to identify its best free set quickly?

Answer: Thresholding automatically hints the free set!

Range Mapping Theorem:

Fix $\rho = (1+\epsilon)\lambda$ OPT$_w$:= MinRow(A^{kc}, w)

$w_\rho(e) := \min\{w(e), \rho\}$

1. If OPT$_w \in [\lambda, (1+\epsilon)\lambda)$ then minimum cut w.r.t. $w_\rho \in [\frac{k\rho}{1+\epsilon}, k\rho)$
2. If a cut C having $w_\rho(C) \leq k\rho$ then $\frac{w(C \backslash F^*)}{k - |F^*|} < (1+\epsilon)\lambda$

$F^* := \{e \in C : w(e) \geq \rho\}$

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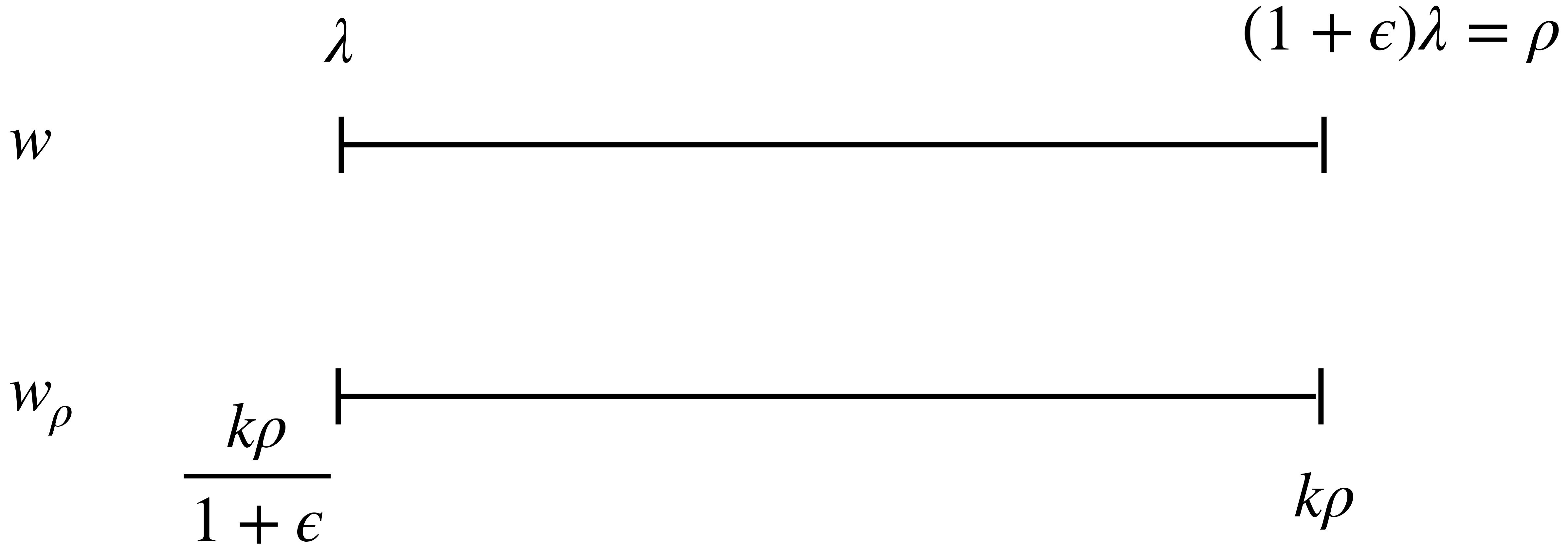

Algorithm:

Initialize w_ρ

Run the dynamic mincut algorithm from [CQ'17] in w_ρ

- List mincut in $[\frac{k\rho}{1+\epsilon}, k\rho)$
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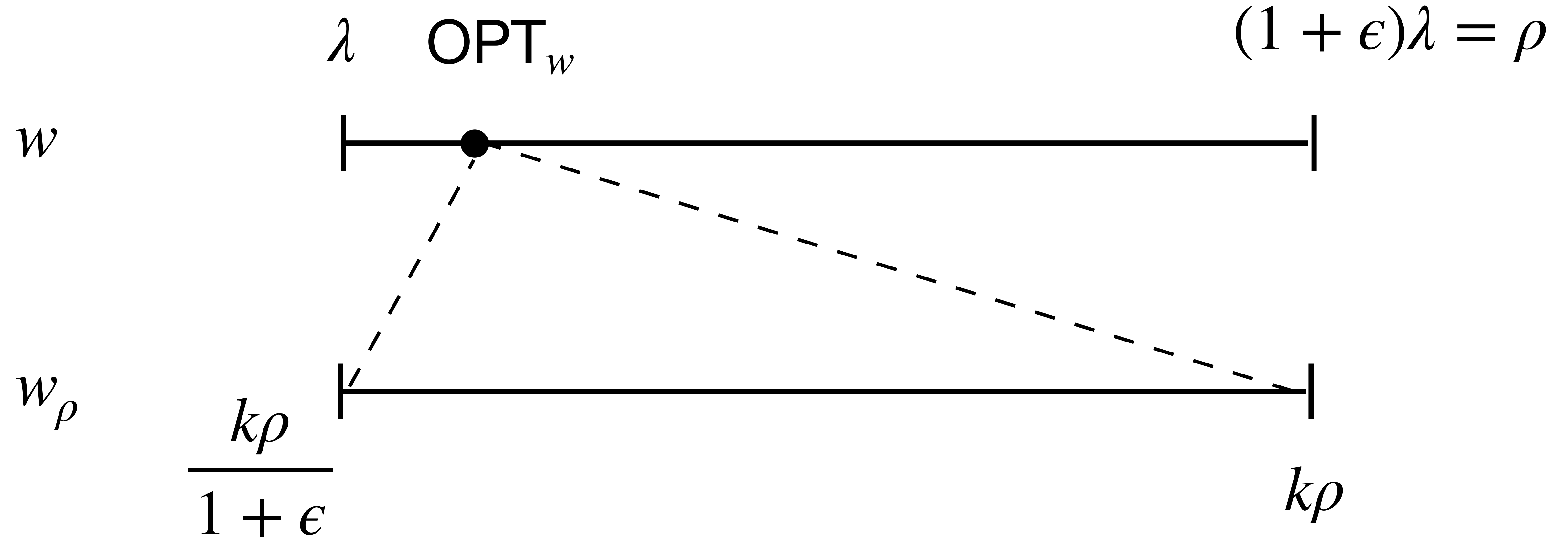
Range Mapping (1)



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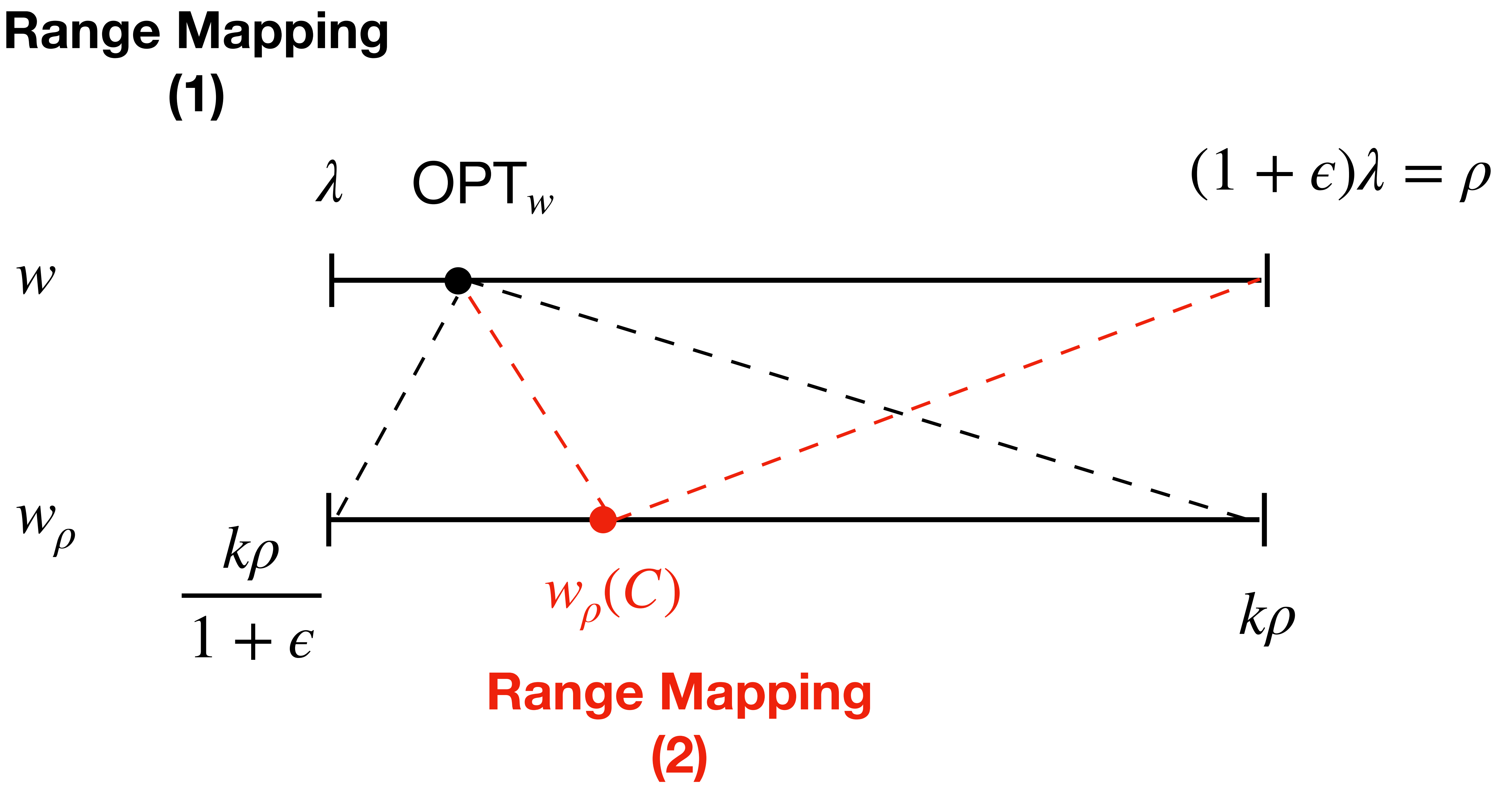

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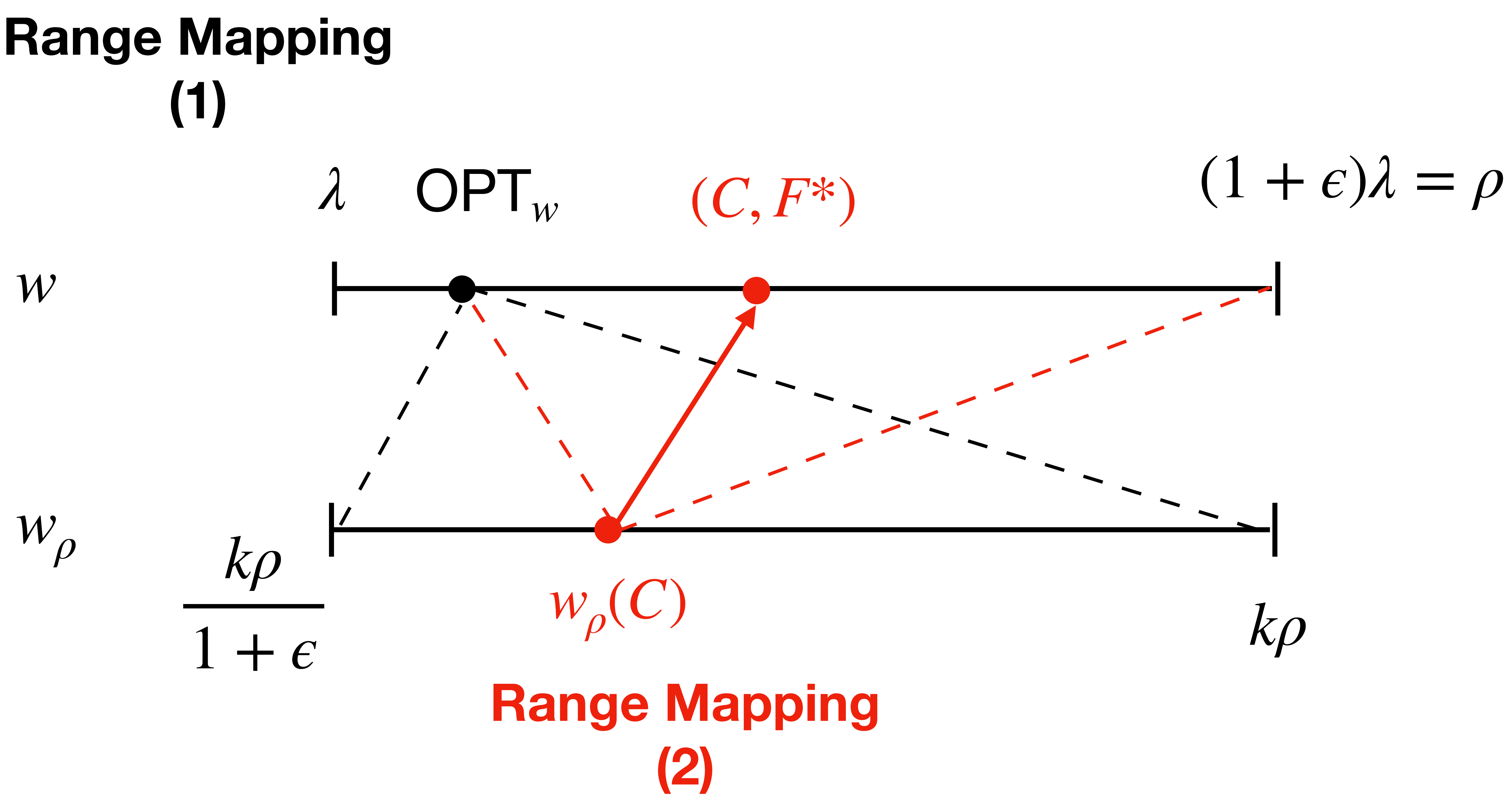

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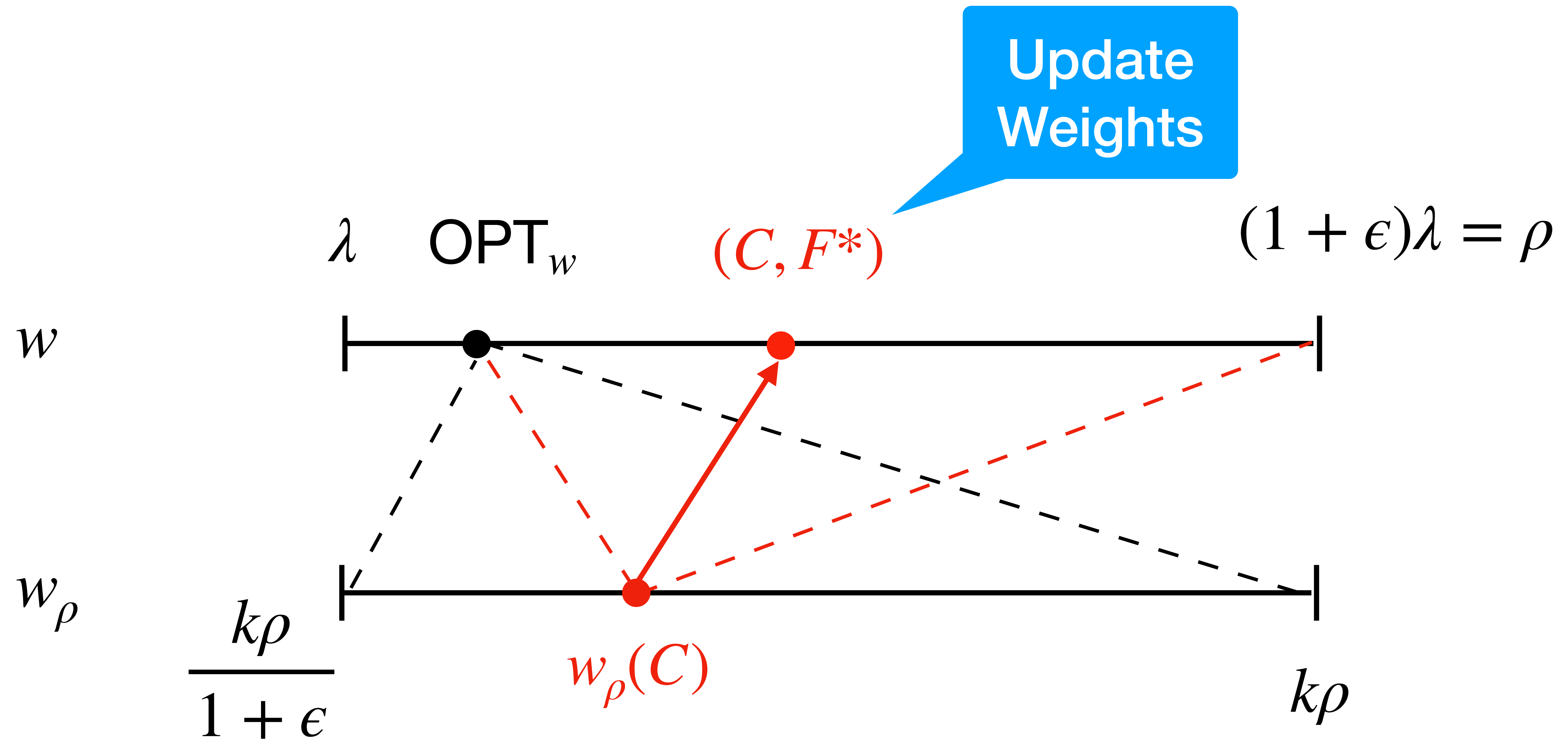

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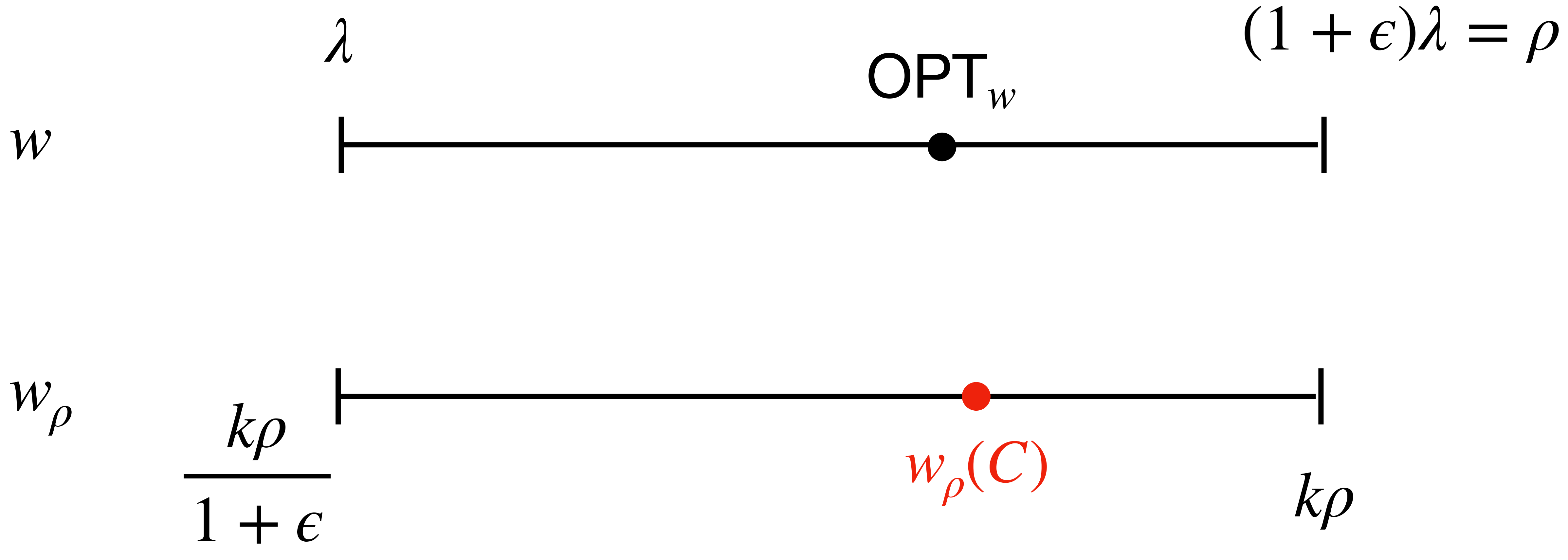

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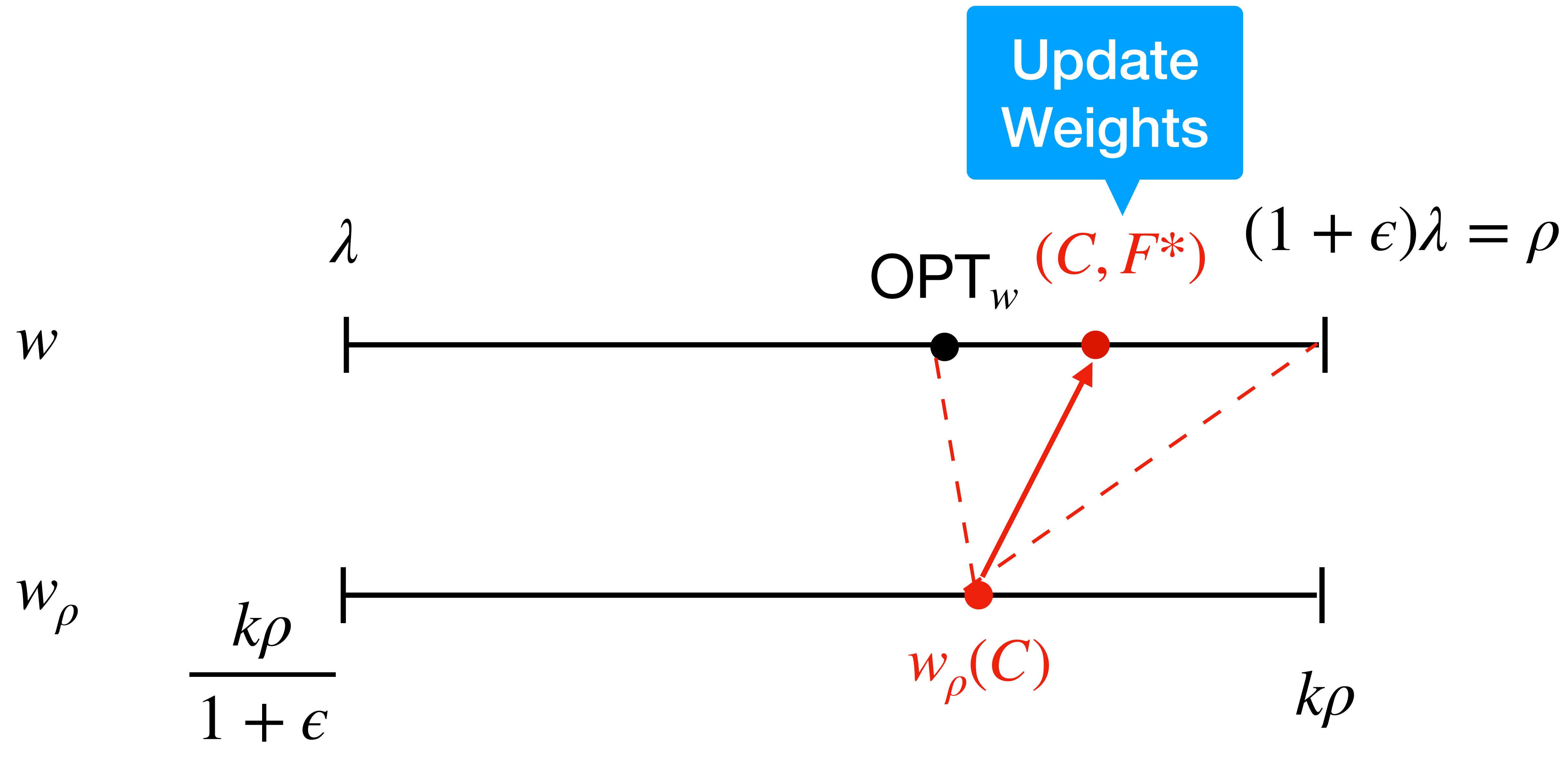
- Update weights and repeat until mincut > $k\rho$

Range Mapping Theorem: Fix $\rho = (1+\epsilon)\lambda$ $\quad \mathrm{OPT}_w := \mathrm{MinRow}(A^{kc}, w)$

$$w_\rho(e) := \min\{w(e), \rho\}$$

1. If $\mathrm{OPT}_w \in [\lambda, (1+\epsilon)\lambda)$ then minimum cut w.r.t. $w_\rho \in [\frac{k\rho}{1+\epsilon}, k\rho)$
2. If a cut C having $w_\rho(C) \le k\rho$ then $\frac{w(C \backslash F^*)}{k - |F^*|} < (1+\epsilon)\lambda$



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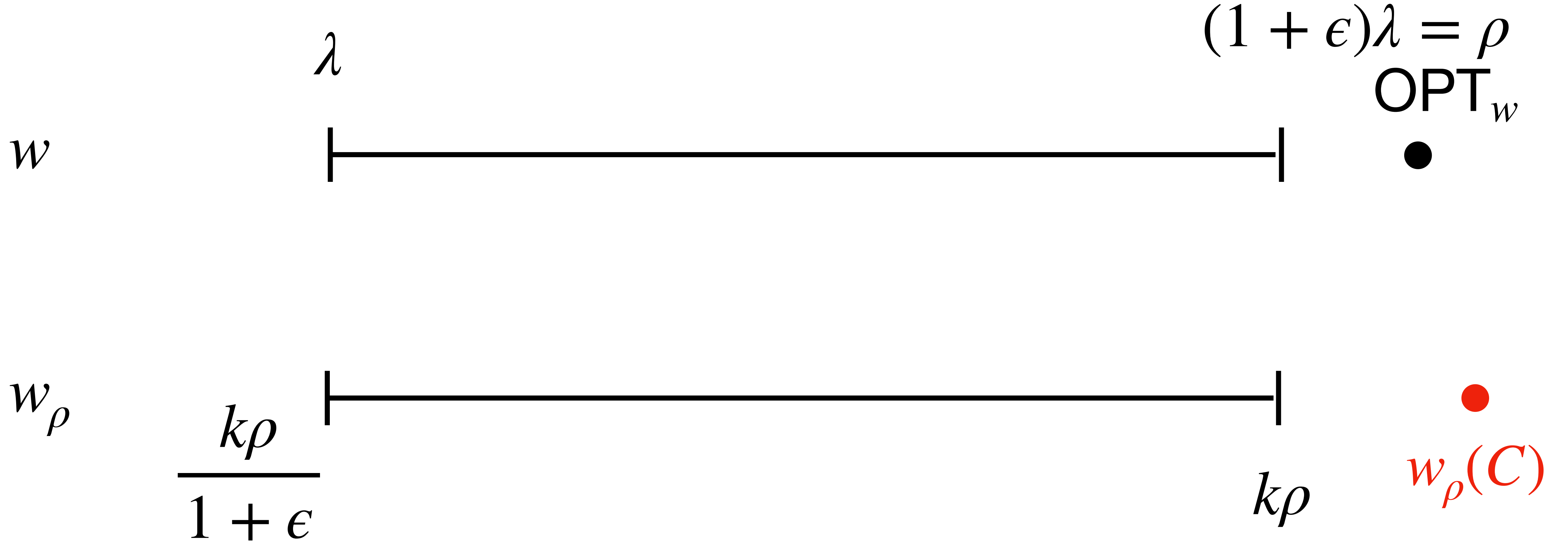

End of the epoch

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Possible to Implement in time $\tilde{O}(m + K_i)$ $\quad K_i =$ # cuts

Open Problems

- Theory of Fast Approximation Algorithms
  - Fast rounding algorithms? Nearly linear time for 2ECSS?
  - Fast LP Solvers for a broader class e.g., SNDP?
  - Can we solve LP with high-accuracy in nearly linear time?
    - Iterative rounding algorithm needs high-accuracy solvers