Structure-guided Local Improvement for Maximum Satisfiability

Vaidyanathan P. R., <u>André Schidler</u>, Stefan Szeider
Algorithms & Complexity Group, TU Wien

MaxSAT
Definition

- We are given

  - a set of hard clauses.

  - a set of soft clauses with weights or unweighted.

- A solution is a variable assignment

  - satisfying all hard clauses.

  - maximising the summed weight of satisfied soft clauses.

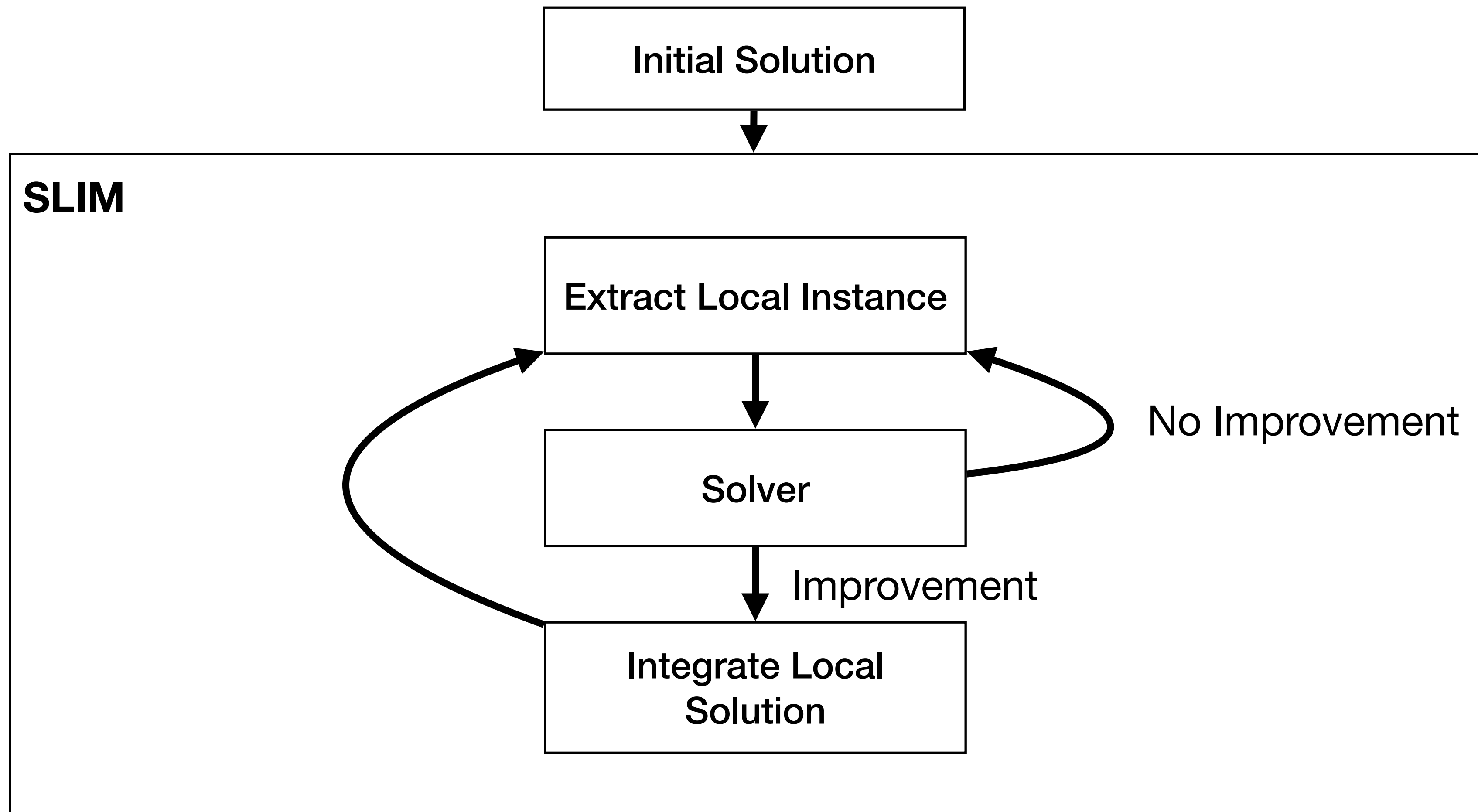 - minimising the cost—the summed weight of unsatisfied soft clauses.

MaxSAT
Incomplete Solvers

- We developed an approach for incomplete MaxSAT solving:

  - Incomplete solvers are anytime solvers.

  - Quick initial non-optimal assignment.

  - Incrementally improve the assignment.

  - No guarantee of optimality.

SAT-based Local Improvement (SLIM)

- Based on a structural representation of the problem.

- Uses a complete solver inside a heuristic algorithm:

  - Starts from an initial feasible solution.

  - Repeatedly extracts a small (local) instance and passes it to the solver.

  - Any improvement for the local instance found by the SAT/MaxSAT/SMT/PB/QBF solver is integrated in the global solution.

- Successfully used for: decision tree minimisation, Bayesian network structure learning, graph coloring, graph decompositions, boolean circuit minimisation,…

SAT-based Local Improvement (SLIM)

SLIM for MaxSAT
Local Instances

- Given: an assignment satisfying all hard clauses.

- Idea:

  1. Split the variables into **fixed** and **free** variables.

  2. Remove from the instance all clauses satisfied by the fixed variables.

  3. Remove all literals from clauses that are unsatisfied given the fixed values.

  4. Perform unit propagation for the remaining instance.

  5. Any improved assignment to this local instance translates to an overall improvement.

  6. Repeat

Example

Initial Assignment: $\{\neg v_1, \neg v_2, \neg v_3, \neg v_4, v_5\}$ Fixed: $\{v_2, v_4\}$ Free: $\{v_1, v_3, v_5\}$

Hard Clauses:

$\neg v_1 \lor \neg v_2 \lor \neg v_3$

$\neg v_1 \lor \neg v_3 \lor v_5$

$v_2 \lor \neg v_3 \lor \neg v_5$

$\neg v_1 \lor v_2 \lor v_4$

$v_2 \lor v_3 \lor \neg v_4$

$\neg v_1 \lor v_2 \lor \neg v_5$

$\neg v_2 \lor v_3 \lor v_4$

$\neg v_1 \lor v_3 \lor v_4$

$v_2 \lor v_3 \lor v_5$

Soft Clauses:

v_1, v_2, v_3, v_4, v_5

Example

Initial Assignment: $\{\neg v_1, \neg v_2, \neg v_3, \neg v_4, v_5\}$ Fixed: $\{v_2, v_4\}$ Free: $\{v_1, v_3, v_5\}$

Hard Clauses:

$\neg v_1 \lor \neg v_2 \lor \neg v_3$

$\neg v_1 \lor \neg v_3 \lor v_5$

$v_2 \lor \neg v_3 \lor \neg v_5$

$\neg v_1 \lor v_2 \lor v_4$

$v_2 \lor v_3 \lor \neg v_4$

$\neg v_1 \lor v_2 \lor \neg v_5$

$\neg v_2 \lor v_3 \lor v_4$

$\neg v_1 \lor v_3 \lor v_4$

$v_2 \lor v_3 \lor v_5$

Hard Clauses:

$\neg v_1 \lor \neg v_3 \lor v_5$

$\neg v_3 \lor \neg v_5$

$\neg v_1$

$\neg v_1 \lor \neg v_5$

$\neg v_1 \lor v_3$

$v_3 \lor v_5$

Soft Clauses:

v_1, v_2, v_3, v_4, v_5

Soft Clauses:

v_1, v_3, v_5

Example

Initial Assignment: $\{\neg v_1, \neg v_2, \neg v_3, \neg v_4, v_5\}$ Fixed: $\{v_2, v_4\}$ Free: $\{v_1, v_3, v_5\}$

Hard Clauses:

$\neg v_1 \vee \neg v_2 \vee \neg v_3$

$\neg v_1 \vee \neg v_3 \vee v_5$

$v_2 \vee \neg v_3 \vee \neg v_5$

$\neg v_1 \vee v_2 \vee v_4$

$v_2 \vee v_3 \vee \neg v_4$

$\neg v_1 \vee v_2 \vee \neg v_5$

$\neg v_2 \vee v_3 \vee v_4$

$\neg v_1 \vee v_3 \vee v_4$

$v_2 \vee v_3 \vee v_5$

Soft Clauses:

v_1, v_2, v_3, v_4, v_5

Hard Clauses:

$\neg v_1 \vee \neg v_3 \vee v_5$

$\neg v_3 \vee \neg v_5$

$\neg v_1$

$\neg v_1 \vee \neg v_5$

$\neg v_1 \vee v_3$

$v_3 \vee v_5$

Soft Clauses:

v_1, v_3, v_5

Hard Clauses:

$\neg v_3 \vee \neg v_5$

$v_3 \vee v_5$

Soft Clauses:

v_3, v_5

Primal Graph
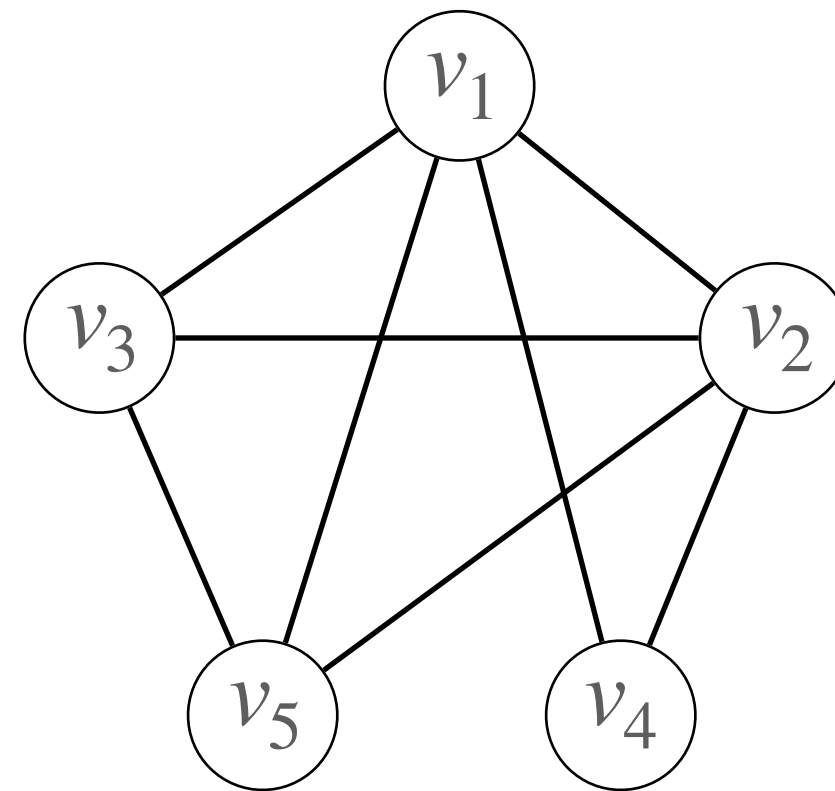
Hard Clauses:

$\neg v_1 \vee \neg v_2 \vee \neg v_3$

$\neg v_1 \vee \neg v_3 \vee v_5$

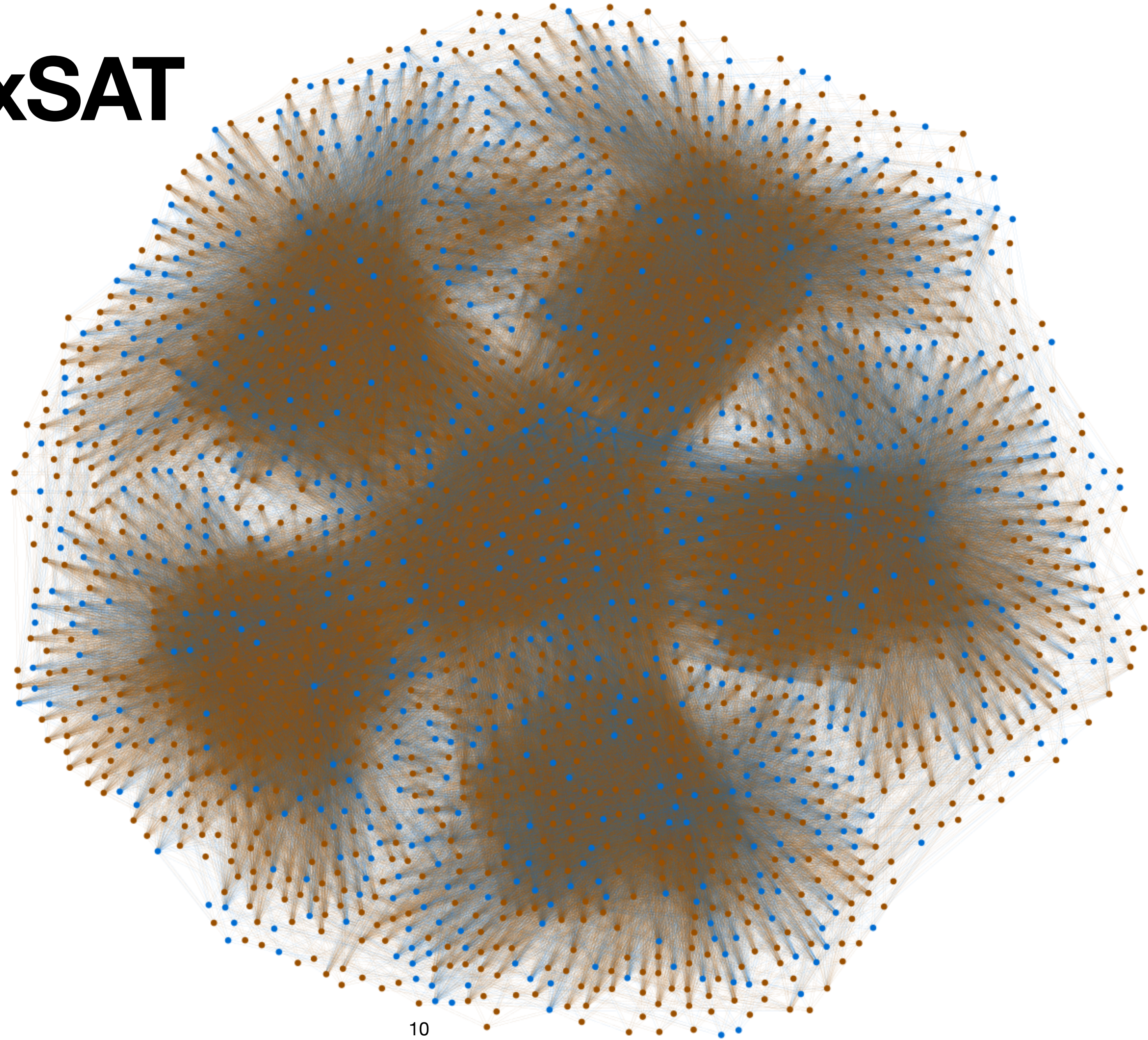$v_2 \vee \neg v_3 \vee \neg v_5$

$\neg v_1 \vee v_2 \vee v_4$

Soft Clauses:

v_1, v_2, v_3, v_4, v_5

LNS for MaxSAT [Hickey & Bacchus IJCAI22]

- Starts with a solution computed by an incomplete solver.

- Assigns weights to variables occurring in soft clauses.

- Chooses the fixed variables according to weighted random sampling.

- Result: beats the best incomplete solver from the 2021 evaluation.

LNS for MaxSAT

SLIM for MaxSAT

- Idea: Use the structure of the instance.

- Incrementally construct the set of free variables:

  - Add free variables from some unsatisfied soft clauses.

  - Repeat until the number of free variables reaches a set limit:

    1. Remove free variables from all clauses.

    2. Add free variables from unsatisfied clauses.

- This construction explores the primal graph.

SLIM for MaxSAT
Example

Initial Assignment:

$\{\neg v_1, \neg v_2, \neg v_3, \neg v_4, v_5\}$

Hard Clauses:

$\neg v_1 \vee \neg v_2 \vee \neg v_3$

$\neg v_1 \vee \neg v_3 \vee v_5$

$v_2 \vee \neg v_3 \vee \neg v_5$

$\neg v_1 \vee v_2 \vee v_4$

$v_2 \vee v_3 \vee \neg v_4$

$\neg v_1 \vee v_2 \vee \neg v_5$

$\neg v_2 \vee v_3 \vee v_4$

$\neg v_1 \vee v_3 \vee v_4$

$v_2 \vee v_3 \vee v_5$

Soft Clauses:

v_1, v_2, v_3, v_4, v_5

SLIM for MaxSAT

Example

Initial Assignment: Free:

$\{ \neg v_1, \neg v_2, \neg v_3, \neg v_4, v_5 \}$ $\{ v_1 \}$

Hard Clauses: Hard Clauses:

$\neg v_1 \lor \neg v_2 \lor \neg v_3$ $\neg v_2 \lor \neg v_3$

$\neg v_1 \lor \neg v_3 \lor v_5$ $\neg v_3 \lor v_5$

$v_2 \lor \neg v_3 \lor \neg v_5$ $v_2 \lor \neg v_3 \lor \neg v_5$

$\neg v_1 \lor v_2 \lor v_4$ $v_2 \lor v_4$

$v_2 \lor v_3 \lor \neg v_4$ $v_2 \lor v_3 \lor \neg v_4$

$\neg v_1 \lor v_2 \lor \neg v_5$ $v_2 \lor \neg v_5$

$\neg v_2 \lor v_3 \lor v_4$ $\neg v_2 \lor v_3 \lor v_4$

$\neg v_1 \lor v_3 \lor v_4$ $v_3 \lor v_4$

$v_2 \lor v_3 \lor v_5$ $v_2 \lor v_3 \lor v_5$

Soft Clauses: Soft Clauses:

v_1, v_2, v_3, v_4, v_5 $v_2, v_3, v_4, \quad v_5$

SLIM for MaxSAT
Example

Initial Assignment:

$\{\neg v_1, \neg v_2, \neg v_3, \neg v_4, v_5\}$

Hard Clauses:

$\neg v_1 \lor \neg v_2 \lor \neg v_3$

$\neg v_1 \lor \neg v_3 \lor v_5$

$v_2 \lor \neg v_3 \lor \neg v_5$

$\neg v_1 \lor v_2 \lor v_4$

$v_2 \lor v_3 \lor \neg v_4$

$\neg v_1 \lor v_2 \lor \neg v_5$

$\neg v_2 \lor v_3 \lor v_4$

$\neg v_1 \lor v_3 \lor v_4$

$v_2 \lor v_3 \lor v_5$

Soft Clauses:

v_1, v_2, v_3, v_4, v_5

Free:

$\{v_1\}$

Hard Clauses:

$\neg v_2 \lor \neg v_3$

$\neg v_3 \lor v_5$

$v_2 \lor \neg v_3 \lor \neg v_5$

$v_2 \lor v_4$

$v_2 \lor v_3 \lor \neg v_4$

$v_2 \lor \neg v_5$

$\neg v_2 \lor v_3 \lor v_4$

$v_3 \lor v_4$

$v_2 \lor v_3 \lor v_5$

Soft Clauses:

$v_2, v_3, v_4, \quad v_5$

Free:

$\{v_1, v_2\}$

Hard Clauses:

$\neg v_3$

$\neg v_3 \lor v_5$

$\neg v_3 \lor \neg v_5$

v_4

$v_3 \lor \neg v_4$

$\neg v_5$

$v_3 \lor v_4$

$v_3 \lor v_4$

$v_3 \lor v_5$

Soft Clauses:

$v_3, v_4, \quad v_5$

SLIM for MaxSAT
Example

Initial Assignment: Free: Free: Free:

$\{\neg v_1, \neg v_2, \neg v_3, \neg v_4, v_5\}$ $\{v_1\}$ $\{v_1, v_2\}$ $\{v_1, v_2, v_3\}$

Hard Clauses:	Hard Clauses:	Hard Clauses:	Hard Clauses:
$\neg v_1 \vee \neg v_2 \vee \neg v_3$	$\neg v_2 \vee \neg v_3$	$\neg v_3$	
$\neg v_1 \vee \neg v_3 \vee v_5$	$\neg v_3 \vee v_5$	$\neg v_3 \vee v_5$	v_5
$v_2 \vee \neg v_3 \vee \neg v_5$	$v_2 \vee \neg v_3 \vee \neg v_5$	$\neg v_3 \vee \neg v_5$	$\neg v_5$
$\neg v_1 \vee v_2 \vee v_4$	$v_2 \vee v_4$	v_4	v_4
$v_2 \vee v_3 \vee \neg v_4$	$v_2 \vee v_3 \vee \neg v_4$	$v_3 \vee \neg v_4$	v_4
$\neg v_1 \vee v_2 \vee \neg v_5$	$v_2 \vee \neg v_5$	$\neg v_5$	$\neg v_5$
$\neg v_2 \vee v_3 \vee v_4$	$\neg v_2 \vee v_3 \vee v_4$	$v_3 \vee v_4$	v_4
$\neg v_1 \vee v_3 \vee v_4$	$v_3 \vee v_4$	$v_3 \vee v_4$	v_4
$v_2 \vee v_3 \vee v_5$	$v_2 \vee v_3 \vee v_5$	$v_3 \vee v_5$	v_5

Soft Clauses: Soft Clauses: Soft Clauses: Soft Clauses:

v_1, v_2, v_3, v_4, v_5 $v_2, v_3, v_4, \quad v_5$ $v_3, v_4, \quad\quad v_5$ $v_4, \quad\quad\quad v_5$

SLIM for MaxSAT
Example

Initial Assignment: Free: Free: Free:

$\{\neg v_1, \neg v_2, \neg v_3, \neg v_4, v_5\}$ $\{v_1\}$ $\{v_1, v_2\}$ $\{v_1, v_2, v_3\}$

Hard Clauses:	Hard Clauses:	Hard Clauses:	Hard Clauses:	Hard Clauses:
$\neg v_1 \vee \neg v_2 \vee \neg v_3$	$\neg v_2 \vee \neg v_3$	$\neg v_3$		$\neg v_1 \vee \neg v_2 \vee \neg v_3$
$\neg v_1 \vee \neg v_3 \vee v_5$	$\neg v_3 \vee v_5$	$\neg v_3 \vee v_5$	v_5	
$v_2 \vee \neg v_3 \vee \neg v_5$	$v_2 \vee \neg v_3 \vee \neg v_5$	$\neg v_3 \vee \neg v_5$	$\neg v_5$	$v_2 \vee \neg v_3$
$\neg v_1 \vee v_2 \vee v_4$	$v_2 \vee v_4$	v_4	v_4	$\neg v_1 \vee v_2$
$v_2 \vee v_3 \vee \neg v_4$	$v_2 \vee v_3 \vee \neg v_4$	$v_3 \vee \neg v_4$	v_4	
$\neg v_1 \vee v_2 \vee \neg v_5$	$v_2 \vee \neg v_5$	$\neg v_5$	$\neg v_5$	$\neg v_1 \vee v_2$
$\neg v_2 \vee v_3 \vee v_4$	$\neg v_2 \vee v_3 \vee v_4$	$v_3 \vee v_4$	v_4	$\neg v_2 \vee v_3$
$\neg v_1 \vee v_3 \vee v_4$	$v_3 \vee v_4$	$v_3 \vee v_4$	v_4	$\neg v_1 \vee v_3$
$v_2 \vee v_3 \vee v_5$	$v_2 \vee v_3 \vee v_5$	$v_3 \vee v_5$	v_5	

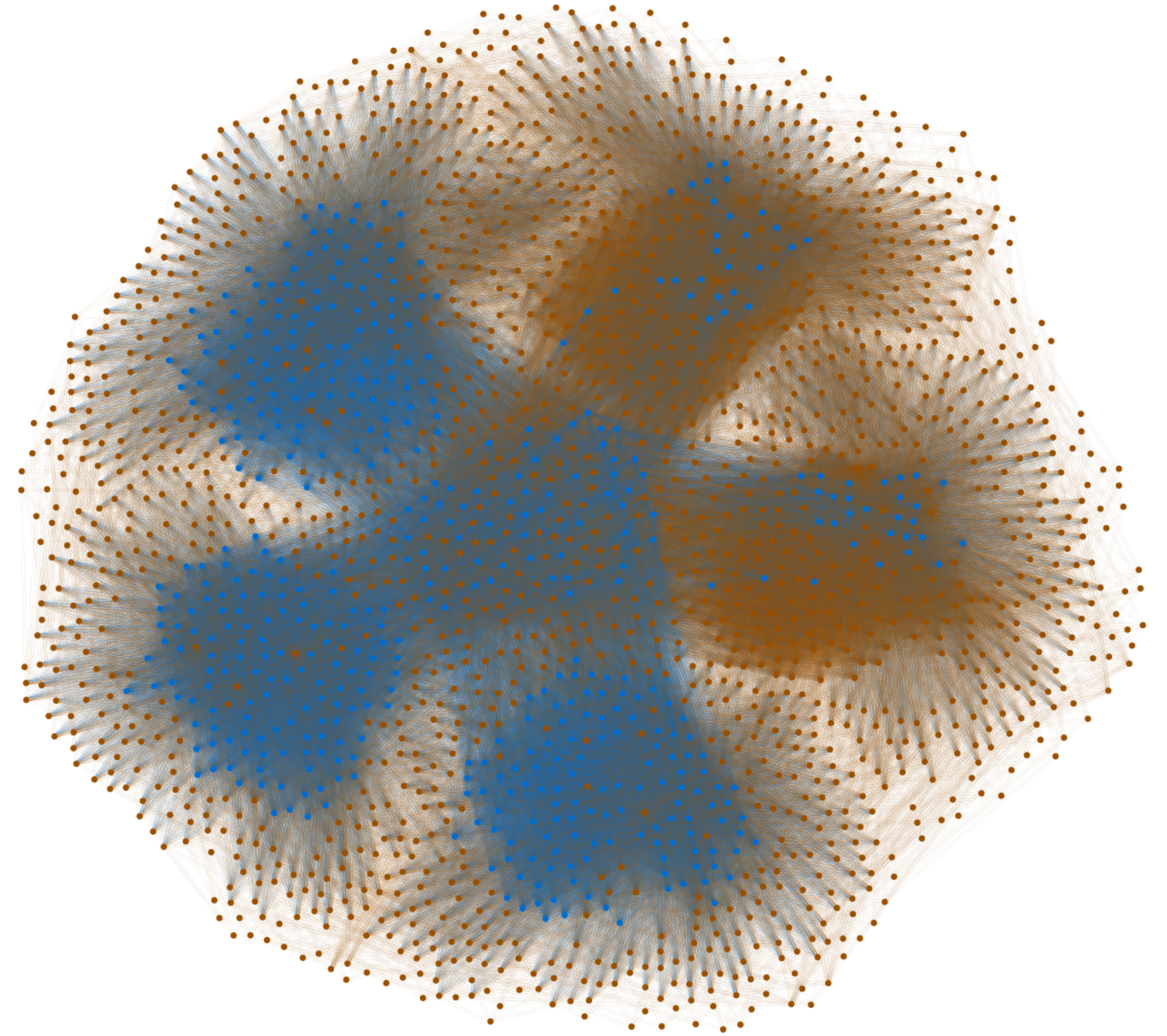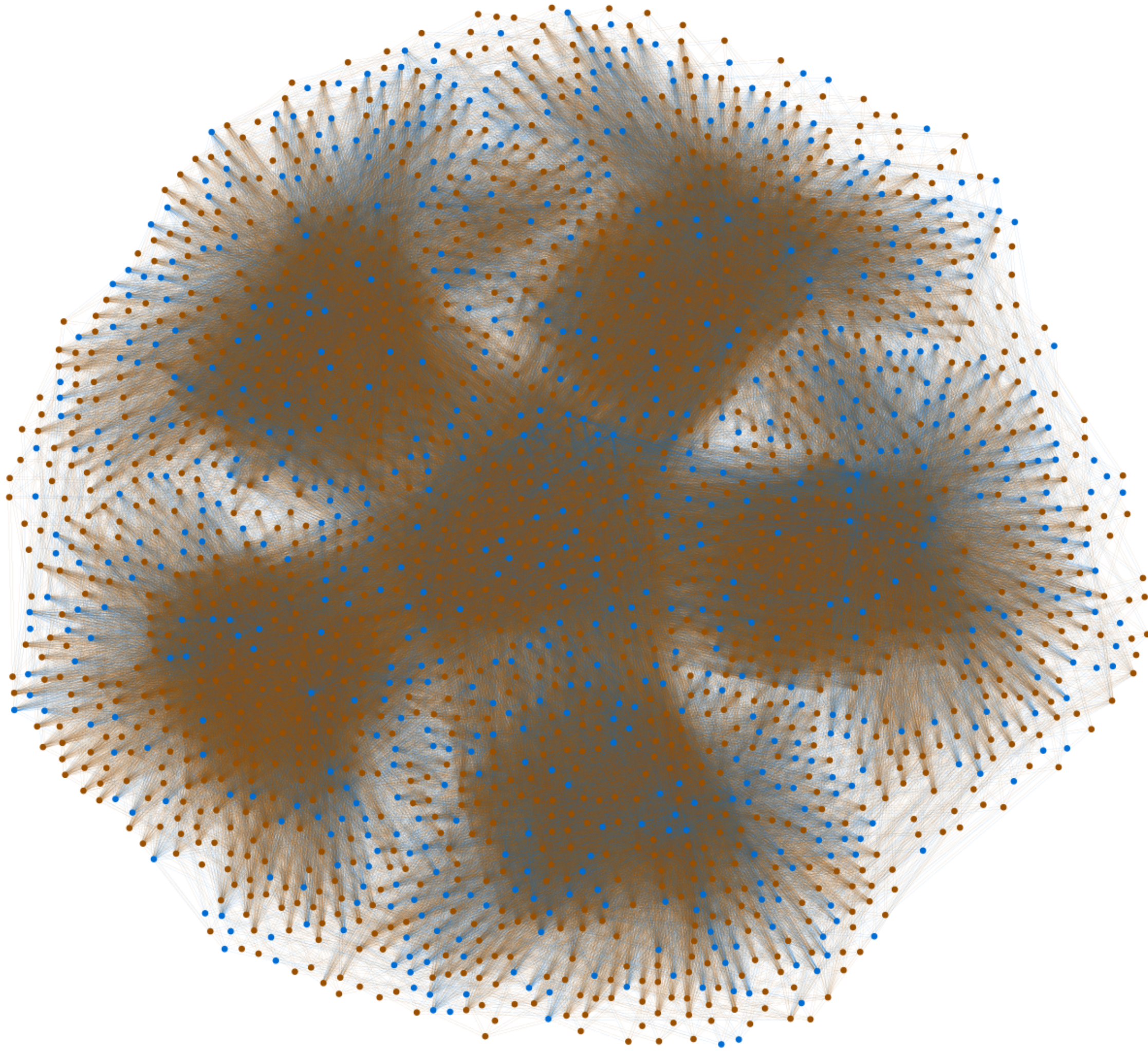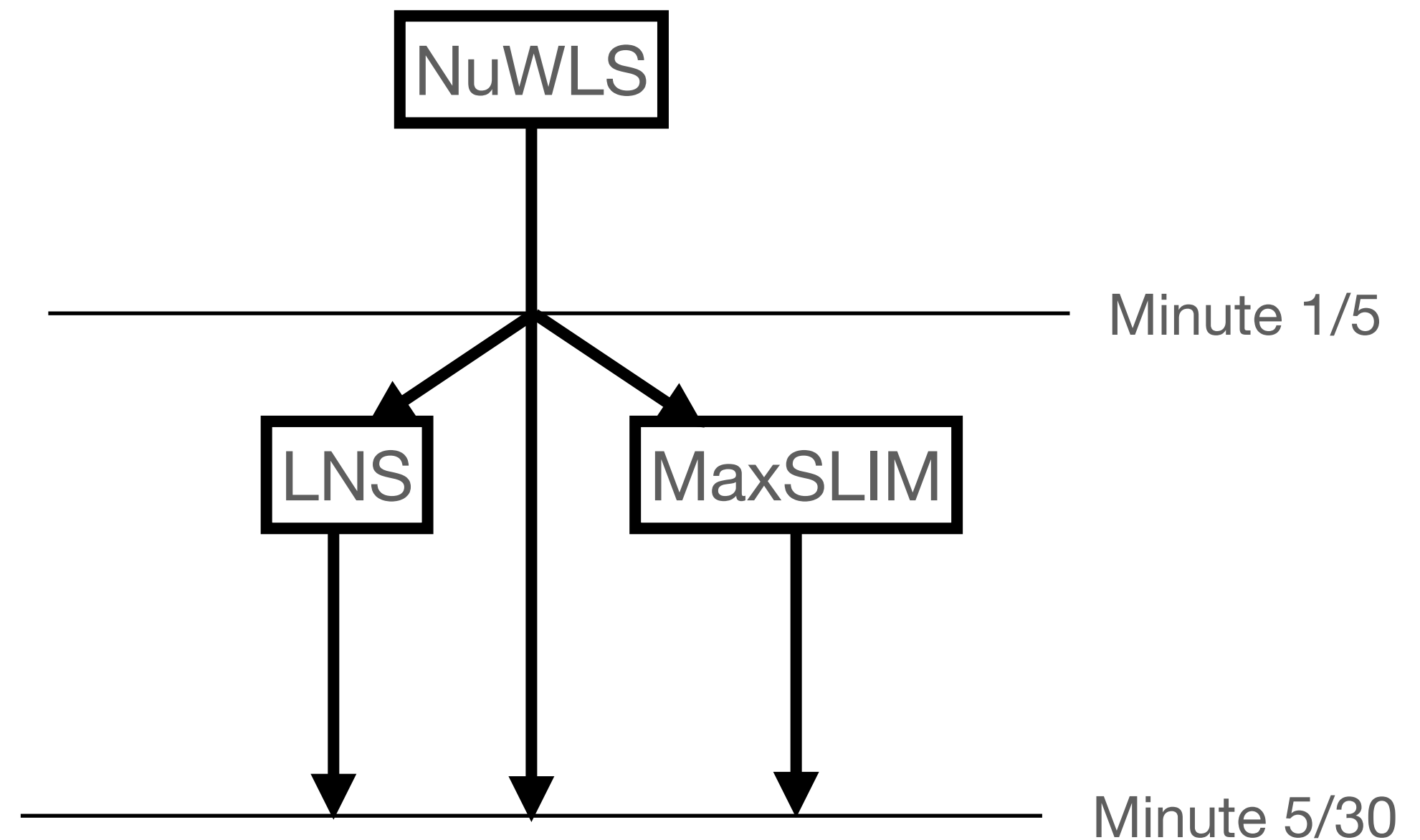| Soft Clauses:	Soft Clauses:	Soft Clauses:	Soft Clauses:	Soft Clauses:
v_1, v_2, v_3, v_4, v_5	$v_2, v_3, v_4,\quad v_5$	$v_3, v_4,\quad v_5$	$v_4,\quad v_5$	v_1, v_2, v_3

SLIM for MaxSAT

SLIM for MaxSAT

Results
Setup

- MaxSAT Evaluation Scoring:

  - Divide best result by solver result.

  - 1 point for best result.

- LNS and MaxSLIM use MaxHS.

- One configuration for MaxSLIM.

Results
Unweighted

Solver	5 Minutes		30 Minutes	
	Score	Improved	Score	Improved
NuWLS	0.873	64	0.891	45
LNS	0.832	81	0.876	49
MaxSLIM	0.844	85	0.886	75

Results
Weighted

Solver	5 Minutes		30 Minutes	
	Score	Improved	Score	Improved
NuWLS	0.868	53	0.885	35
LNS	0.820	75	0.822	89
MaxSLIM	0.836	79	0.856	91

Conclusion

- SLIM is a promising approach for incomplete MaxSAT solving.

- There is still much potential for improvement:

  - There are implementation details that can be improved.

  - Other solver combinations might give better results.

- Easier integration of MaxSAT solvers would have helped.

- Particularly incomplete solvers are hard to integrate.

Open Questions

- Can these ideas be implemented directly in a solver?

- When does this approach work well?

- Is there a better structure for local instances?