Persuasion in Networks

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The Economics of Networks
Simons Institute
November 2022

This work

Information Design for Networks

- Agents in a network take (binary) actions. Two key ingredients:
  - Actions exhibit local strategic complementarities
  - Uncertain state of the world impacts payoffs

$$u_i(a_i, a_{-i}) = a_i(T + \sum_j g_{ij} a_j)$$

- Designer chooses a public signaling mechanism ($S = g(T)$) to maximize expected activity ($\mathbb{E}[\sum_i a_i]$)

Application: How to (publicly) signal product quality to influence purchase decisions?

- Higher payoff from consuming the same product as peers
- Disutility from consuming low quality product
- Objective: maximize sales

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Research questions

Information Design for Networks

- What are the optimal information structures?
- How do they depend on the network structure?
- Persuasion when only limited network information is available?
- Which networks are more amenable to persuasion?

Main contributions

Optimal public signals characterized in terms of graph cores

- Set of possible signal realizations = set of distinct cores
- When the signal realization is k, the k-core takes action 1

Optimal mechanism exhibits a double-interval structure:

- A convex programming formulation + an algorithm to construct the optimal mechanism
- Applicable (well) beyond network persuasion settings See C. and Strack (2022).

Asymptotically optimal mechanisms for large random networks

Degree assortativity makes networks more amenable to persuasion

Related Literature

- Bayesian Persuasion/Information Design: Brocas and Carrillo (2007); Rayo and Segal (2010); Kamenica and Gentzkow (2011); Bergemann anad Morris (2016); Bhaskar et al. (2016); Gentzkow and Kamenica (2017); Dworczak and Martini (2019); Arieli et al. (2020); Kleiner et al. (2020); C. and Strack (2022).

- Games and Networks: Jackson and Wolinsky (1996); Bala and Goyal (2000); Ballester, Calvó-Armengol, Zenou (2006); Galeotti, Goyal, Jackson, Vega-Redondo, Yariv (2010); Candogan, Bimpikis, Ozdaglar (2012); Bloch and Querou (2013); Bramoulle, Kranton, D'Amours (2014); Fainmesser and Galeotti (2015).

- Persuasion in Networks: C. and Drakopoulos (2017), Egorov and Sonin (2019), Galperti and Perego (2019).

Outline

1. Model and structure of optimal mechanisms
2. Large random networks
3. Impact of the network structure
4. Conclusions

Model

Unweighted social network $G = (V, E)$ w/ adjacency matrix $[g_{ij}]_{i,j \in V}$

Each agent $i \in V$ takes a binary action $a_i \in \{0, 1\}$. Payoff:

$$u_i(a_i, a_{-i}) = a_i(T + \sum_j g_{ij} a_j)$$

- $T \sim F$ is the state of the world. It belongs to interval $\mathcal{T} \subset \mathbb{R}$
  - F is continuous and strictly increasing on $\mathcal{T}$
- Agents do not observe T prior to taking action.

Designer commits to a public signaling mechanism that shares an informative signal S with all agents, once T realized

- Objective: maximize total activity $\mathbb{E}[\sum_i a_i]$

Research question: Optimal public signaling mechanism?

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Sender-preferred equilibria

- If multiple eq., focus on the sender-preferred (SP) one
- Signal realization $S = s$ induces a supermodular game with payoffs: $\bar{u}_i(a_i, a_{-i}) = a_i \left(\mathbb{E}[T|S = s] + \sum_j g_{ij} a_j \right)$
- Largest equilibrium: k-core, with $k = \lceil -\mathbb{E}[T|S = s] \rceil$
  - k-core: maximal induced subgraph where all nodes have degree $\geq k$
- Lemma: In a (SP) eq. for any signal realization, a core of the network takes action 1.
- Corollary: In an optimal mechanism, signal realizations = cores, and for signal realization k, the k-core finds it optimal to take action 1.

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Designing optimal public mechanisms

To obtain an optimal mechanism partition $\mathcal{T}$ such that each partition element corresponds to a core

$\mathcal{T}$:

An optimization formulation:

$$\max_{partitions} \sum_k \mathbb{P}(T \in Partition_k) r_k$$

$$s.t. \quad \mathbb{E}[T | T \in Partition_k] \geq -k, \quad \forall k$$

Notation: r_k denotes the cardinality of the k-core.

Fundamental difficulty: The set of all possible partitions is large

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Idea: Formulation over posterior mean distributions

We can pose the designer's problem as an optimization problem over posterior mean distributions consistent with the prior.

- G is a valid posterior mean distribution iff

$$\int_{\omega}^{\infty} G(z)dz \geq \int_{\omega}^{\infty} F(z)dz, \qquad \text{(MPC)}$$

  with equality at the smallest point in the support.
- It suffices to restrict attention to discrete distributions where # mass points = # of distinct cores.
  - A restatement ($z_k = m_k p_k$):

$$\sum_{k \leq \ell} z_k \leq \int_{1-\sum_{k\leq \ell} p_k}^{1} F^{-1}(x)dx$$

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A two step approach

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- Construct a mechanism $\pi^\star$ consistent w/ an optimal solution $\{p_k^\star, z_k^\star\}$. $\pi^\star$ is optimal.
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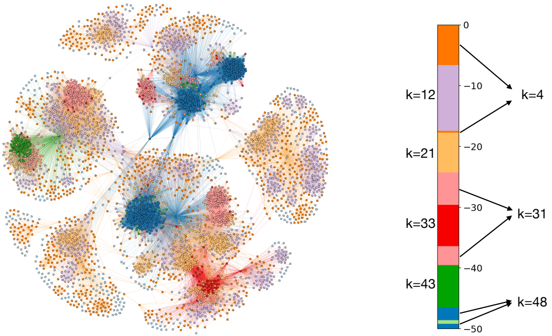
Optimality of Double Intervals

Theorem: Optimal mechanism admits a double interval structure, and can be obtained using the convex program and a recursive algorithm.



- Optimal mechanism for a Facebook subnetwork w/ 4039 nodes

Intuition

Why not > 2 intervals for some signal realizations?

- Start with > 2 intervals for some signal realization.
- Possible to modify the partition in a way that yields 2 intervals and preserves
  - the probability of sending each signal
  - and the associated posterior means.
- This reasoning does not work for the initial partition:

$\mathcal{T}$: 

- A technical lemma rules out the optimality of such partitions.

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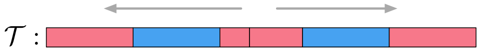

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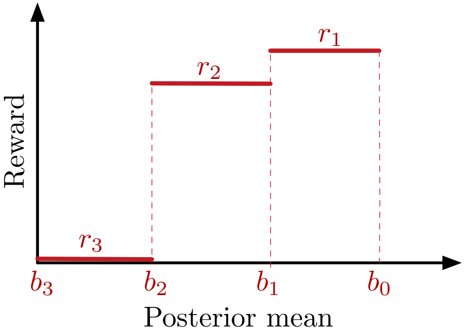

Pooling low and high states expands the set of implementable posterior mean distributions, yielding larger payoff to the designer.

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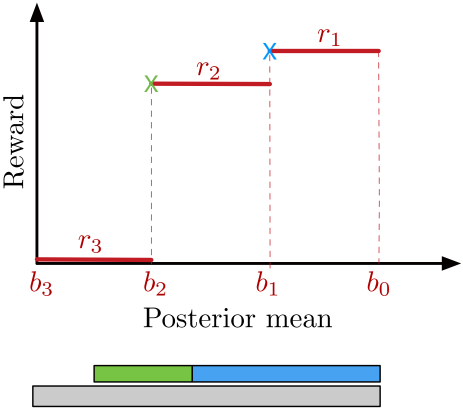

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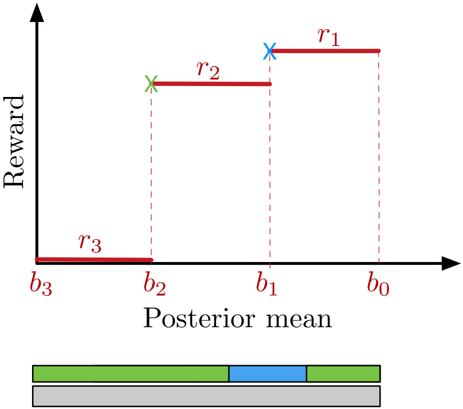

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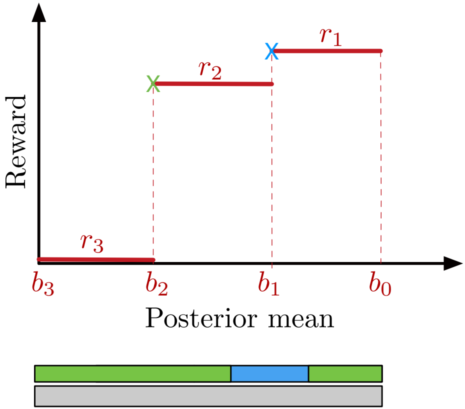
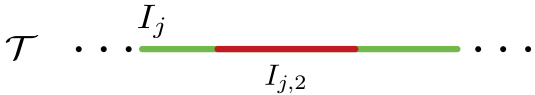


Pooling low and high states expands the set of implementable posterior mean distributions, yielding larger payoff to the designer.

Constructing an optimal partition

- Partitioning problem decouples over subintervals of the state space with at most two mass points each.
- It is straightforward to partition each interval into two partition elements yielding the double interval structure:



$$\mathbb{P}(T \in I_{j,2}) = p_{j,2}$$
$$\mathbb{E}[T | T \in I_{j,2}] = z_{j,2}/p_{j,2}$$

- Both the decoupling result, and the partition structure generalize to richer settings (laminar partitions).
- Question: Multi-dimensional analogue?

Optimality of Double Intervals: Implications

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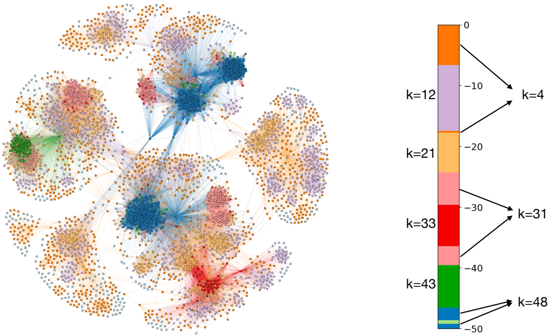

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Partial information about the network

- What if the designer does not know the network perfectly?
- The approach relies on knowing only the cardinalities of cores. If cores can be characterized, similar approach can still be used.

- (Approximate) characterization of cores when limited information about network is available?
  - It turns out that knowing only the degree dist. suffices!

Partial information about the network

A random graph model:

- Let $\{d_i^{(n)}\}_{i=1}^n$ be a degree sequence with n nodes.
- G_n is a uniform draw from set of networks w/ this deg. sequence
- $\rho := \{\rho_k\}$ is a degree distribution s.t. $\frac{|\{i|d_i^{(n)}=k\}|}{n} \to \rho_k$.

Notation: Let $B_{li}(\theta) := \binom{l}{i}\theta^i(1-\theta)^{l-i}$. Define:

- $h^k(x) := \sum_{i=k}^{\infty} \sum_{l=i}^{\infty} i\rho_l B_{li}(x)$
- $h_1^k(x) := \sum_{i=k}^{\infty} \sum_{l=i}^{\infty} \rho_l B_{li}(x)$
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Large networks and Cores – II

Theorem (Janson and Luczak 2007):

1. If $x_k = 0$ then $r_k(n) = o_P(n)$.
2. If $x_k > 0$ and $\lambda x^2 < h^k(x)$ for $x \in (x_k - \epsilon, x_k)$ and some $\epsilon > 0$, then $\frac{r_k(n)}{n} \xrightarrow{p} h_1^k(x_k)$.

Implications:

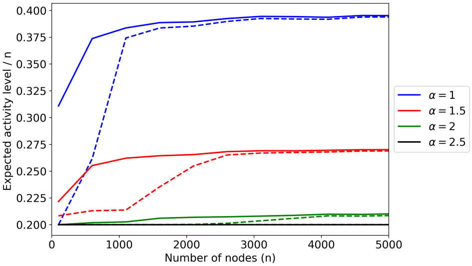
- Set $\hat{r}_k = h_1^k(x_k)$. Theorem implies that $\hat{r}_k$ fraction of nodes in the k-core (asymptotically)
- Let $\bar{k}$ denote the largest k for which $x_k > 0$. Replace the objective of (OPT) with $\sum_{k=0}^{\bar{k}} p_k \hat{r}_k$.
- Denote by $\hat{\pi}$ the public mechanism constructed via the optimal solution of this problem, and the algorithm.

Asymptotically optimal mechanisms

$\pi(n)$: optimal mechanism that uses complete network information.
$\mathcal{A}(\pi, G)$: Designer's payoff under mech. π in network G.

Theorem: $\frac{\mathcal{A}(\hat{\pi}, G_n)}{\mathcal{A}(\pi(n), G_n)} \xrightarrow{p} 1.$

Takeaway: Degree distribution suffices for constructing asymptotically optimal mechanisms.



Solid line: $\pi(n)$, dashed line: $\hat{\pi}$. $\rho_l = c\frac{1}{l^\alpha}$ for $l \in \{d_{min}, \ldots, d_{max}\}$.

Network Structure and Persuasion

- Are some networks more amenable to persuasion than others?
- What is the impact of the network structure on the payoff of the designer?

- We focus on:
  - Role of assortativity
  - Role of the degree sequence

Role of Assortativity

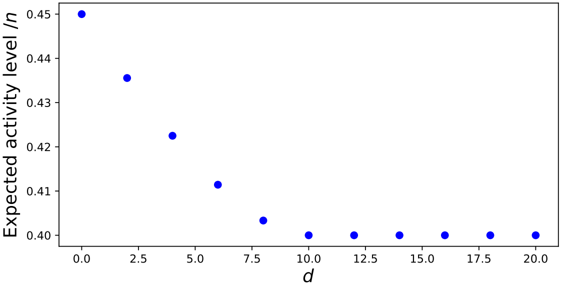
- Consider G^1, G^2 consistent with degree seq. $\{d_i\}$, and the corresponding opt. mech. π^1 and π^2.
  - Edges are wired differently in G^1, G^2. When is the designer's payoff larger?

- Assortativity coefficient: Pearson correlation coef. of degree between pairs of linked nodes.
- Perfectly assortative (corr$= 1$): Neighbors of degree d agent also have degree d.

Theorem: If G^1 is perfectly assortative, then $\mathcal{A}(\pi^1, G^1) \geq \mathcal{A}(\pi^2, G^2)$.

Role of Assortativity – II

Let $d_H = 40, d_L = 20$ and 500 nodes with each degree. Suppose d connections are to the agents of the opposite type. Consider a uniform draw from set of all such networks.



Takeaway: Assortative networks are more amenable to persuasion.

Role of the Degree Sequence

- Consider deg. seq. $d^1 = \{d_i^1\}$, $d^2 = \{d_i^2\}$, and corresponding networks G^1 and G^2 and opt. mech. π^1, π^2.
- Assume $d^1 \succeq d^2$, i.e., $\forall k$ we have $|\{i | d_i^1 \geq k\}| \geq |\{i | d_i^2 \geq k\}|$.
  - (After relabeling) each node has larger degree under d^1.
- Intuitively, in G^1 network externalities are stronger, and hence the designer should have a larger payoff.

Theorem: $\mathcal{A}(\pi^1, G^1) \geq \mathcal{A}(\pi^2, G^2)$ for perfectly assortative G^1, G^2.

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Role of the Degree Sequence

- Consider deg. seq. $d^1 = \{d_i^1\}$, $d^2 = \{d_i^2\}$, and corresponding networks G^1 and G^2 and opt. mech. π^1, π^2.
- Assume $d^1 \succeq d^2$, i.e., $\forall k$ we have $|\{i | d_i^1 \geq k\}| \geq |\{i | d_i^2 \geq k\}|$.
  - (After relabeling) each node has larger degree under d^1.
- Intuitively, in G^1 network externalities are stronger, and hence the designer should have a larger payoff.

Theorem: $\mathcal{A}(\pi^1, G^1) \geq \mathcal{A}(\pi^2, G^2)$ for perfectly assortative G^1, G^2.

The result is not necessarily true w/o perfect assortativity!

Role of the Degree Sequence – II

- Suppose $0.5 - \zeta$ fraction of nodes have degree $d_L = 10$, ζ fraction have degree $d_M = 20$, 0.5 fraction have degree $d_H = 50$.
- The deg. sequence for larger ζ "dominates".
- Consider a uniform draw from the set of all such networks.

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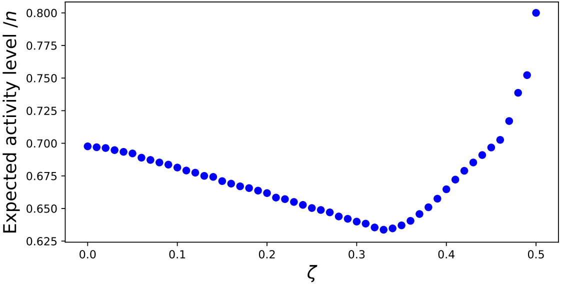
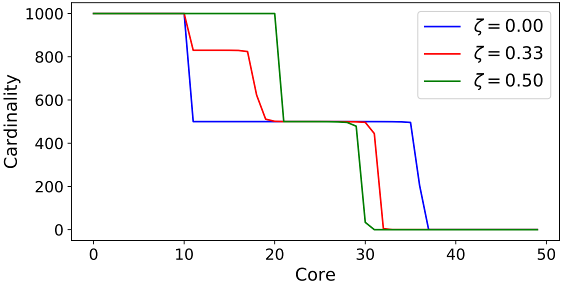


Role of the Degree Sequence – II

Intuition: As ζ increases cardinalities of k-cores for $k \leq 20$ increase and cardinalities of k-cores for $k > 20$ decrease.



Conclusions

Optimal public signals characterized in terms of graph cores

- Set of possible signal realizations = set of distinct cores
- When the signal realization is k, the k-core takes action 1

Optimal mechanism exhibits a double-interval structure:

- A convex programming formulation + an algorithm to construct the optimal mechanism
- Applicable (well) beyond network persuasion settings

Asymptotically optimal mechanisms for large random networks

Degree assortativity makes networks more amenable to persuasion

Questions?