Read-once branching programs as proof lines

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Semantic proof systems

- Let $\varphi = \bigwedge_{i \in I} C_i$ be an unsatisfiable CNF formula.
- Proof lines: Boolean predicates represented somehow:
  - **Resolution**: clauses $(x \vee y \vee \neg z)$
  - **Cutting planes**: linear inequalities with integer coefficients $x - 2y + z \geq 2$
  - **Th(k)**: degree k inequalities with integer coefficients $2xy - yzt + x \geq 3$
  - **Res($\oplus$)**: disjunctions of linear equalities over $\mathbb{F}_2$ $(x + y = 1) \vee (x + z + t = 0) \vee (z = 1)$
- Semantic rule: $\frac{D_1, D_2}{D_3}$ if $D_1, D_2 \models D_3$.
- Semantic refutation of φ: $D_1, D_2, \ldots, D_s$ such that
  - $D_s \equiv 0$
  - D_i either represents a clause of φ or $\frac{D_j, D_k}{D_i}$, where $j, k \leq i$.
- Length: s. Size: $\sum_{i=1}^{s} |D_i|$.

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On lower bounds for semantic proof systems

- If proof lines are too strong, there are upper bounds for all formulas:
  - **CNF formulas**: every UNSAT CNF has a short refutation.
  - **Semantic PC over reals**: every UNSAT 3CNF has a short refutation.
    - $(x \vee y \vee \bar{z}) \wedge (x \vee \bar{y} \vee t) \wedge \dots$
    - $xy(1-z) + x(1-y)t + \dots = 0$, $(x^2-x)^2 + (y^2-y)^2 + \dots = 0$
- [Krajícek, 1995] If proof lines have small deterministic communication complexity, then CliqueColoring is hard.
  - Resolution, CP*
- [Beame, Pitassi, Segerlind, 2007] If proof lines have small randomized communication complexity, then lifted Tseitin formulas are hard for tree-like refutations.
  - Tree-like Th(k), tree-like Res($\oplus$).

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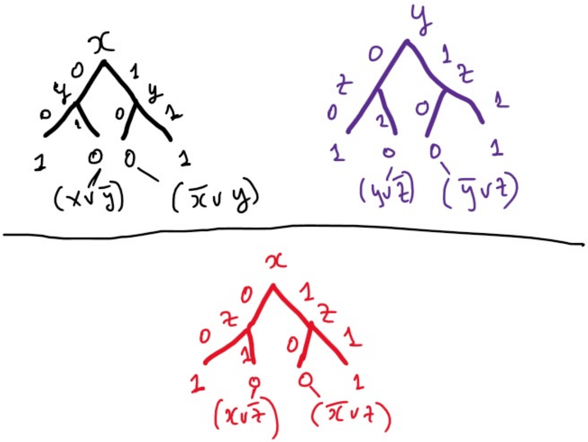
Reasoning by decision trees

Prop. Semantic calculus of decision trees is polynomially equivalent to Resolution.



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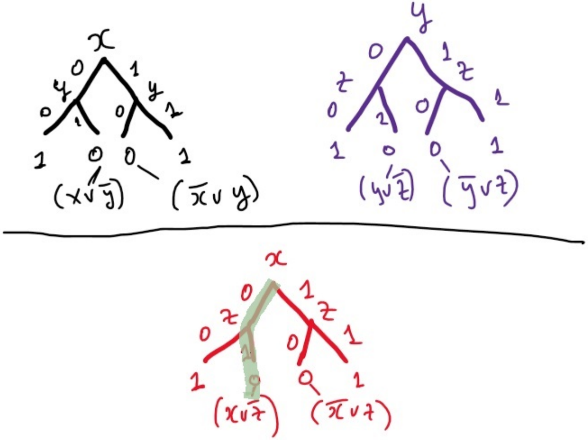

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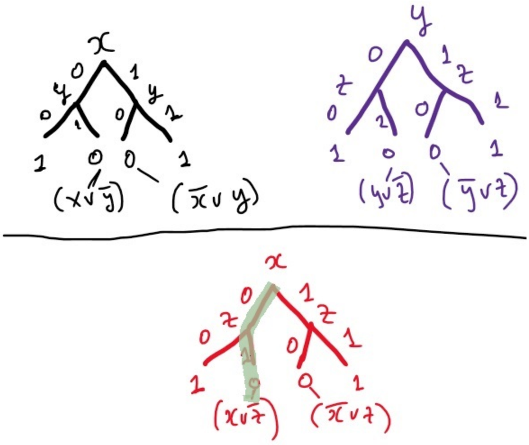

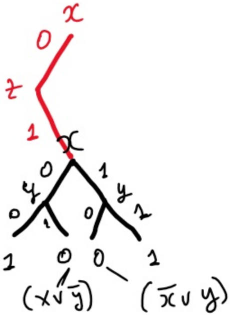

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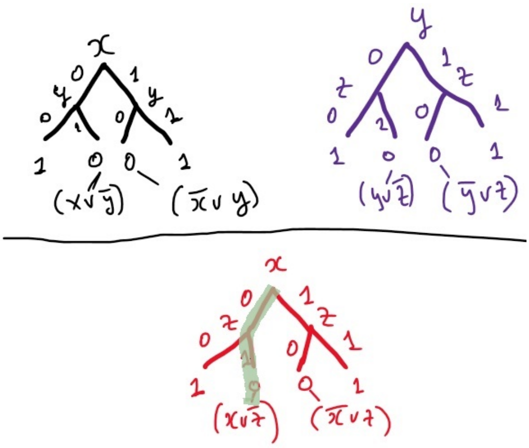

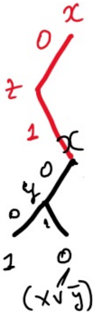

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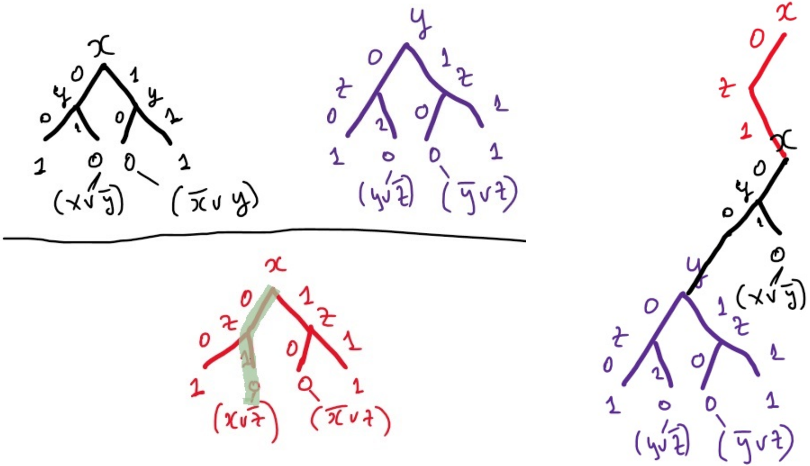

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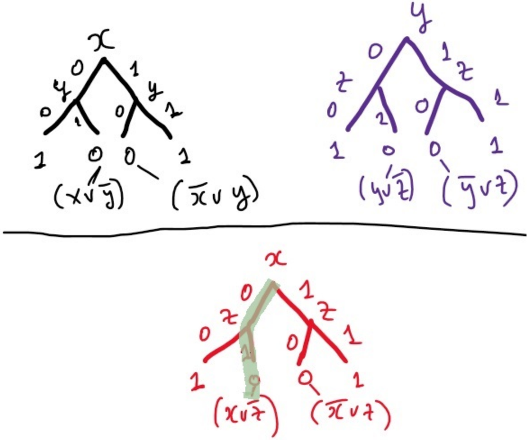
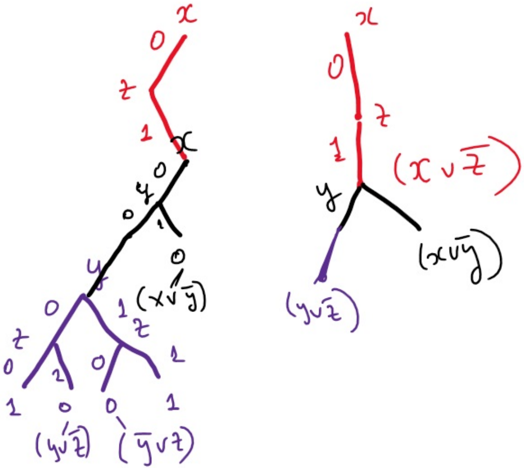
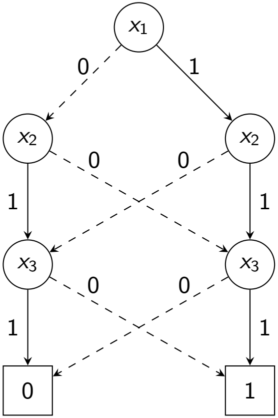




Branching programs



- 1-BP: every path contains different variables.
- OBDD: in all paths variables appear in the same order
- There are small OBDD-representations of clauses, parities and linear inequalities with small coefficients.
- Binary operations for OBDDs in the same order can be computed in polynomial time.
- If partition agrees with the order, then communication complexity of an OBDD of size S is at most $\lceil \log S \rceil + 1$.

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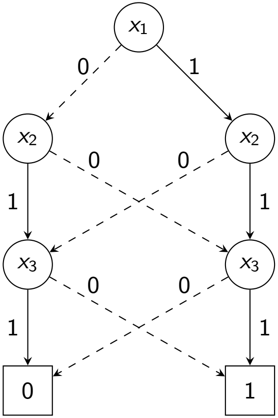

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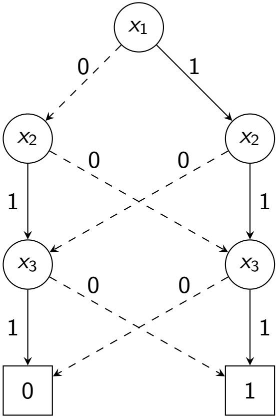

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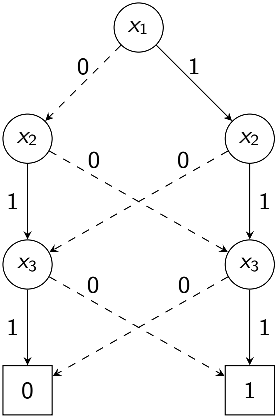

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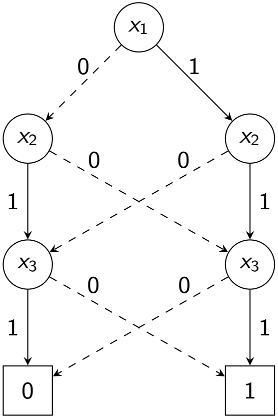

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OBDD-proofs

- [Atserias, Kolaitis, Vardi, 2004] OBDD-based proof systems.
- $\varphi = C_1 \wedge C_2 \wedge \cdots \wedge C_t$ is unsatisfiable CNF.
- Choose order π; every C_i is represented as π-ordered OBDD.
- Rules:
  - Conjunction rule ($\wedge$): $\frac{D_1^\pi, D_2^\pi}{(D_1 \wedge D_2)^\pi}$
  - Weakening rule (w): $\frac{D^\pi}{D_1^\pi}$ if $D \models D_1$.
  - Projection rule ($\exists$): $\frac{D^\pi}{\exists x D^\pi}$
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  - Reordering rule (r): $\frac{D_1^{\pi_1}}{D_2^{\pi_2}}$ if $D_1^{\pi_1} \equiv D_2^{\pi_2}$
- Goal: to derive a constant false OBDD.
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    $$\exists x \begin{cases} x + y + z = 1 \\ x + t + f = 0 \end{cases} \iff y + z + t + f = 1.$$
  - OBDD($\wedge, \exists$) simulates and strictly stronger than resolution:
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  - [Chen, Zhang 2009] Short proof of the pigeonhole principle
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- [Krajicek, 2008] $2^{n^{\Omega(1)}}$-lower bound for dag-like $\mathrm{OBDD}(\wedge, \mathrm{w})$-proofs:
  - $\varphi(x)$ is a formula hard for one order π;
  - $\mathcal{K}(\varphi) = (\sigma \text{ encodes a permutation}) \wedge \varphi(\sigma(x))$;
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  - Orification: $\varphi(x_1, \ldots, x_n) \mapsto \varphi^{\vee_m} = \varphi(\bigvee_{i=1}^m y_{1,i}, \ldots, \bigvee_{i=1}^m y_{n,i})$.
  - $\mathcal{S}(\varphi) = \bigwedge_{\sigma \in \Pi} ((z \text{ encodes } \sigma) \to \varphi^{\vee_m}(\sigma(y)))$, where Π is a small family of 2-independent permutations.
  - $\mathrm{OBDD}(\wedge, \mathrm{w})$ does not simulate $\mathrm{Res}(O(\log n))$.
- [Buss, I., Knop, Sokolov, 2018] Reordering rule makes proof systems stronger.
  - $\mathcal{S}$(Clique-Coloring) separates $\mathrm{OBDD}(\wedge, \mathrm{w}, \mathrm{r})$ and $\mathrm{OBDD}(\wedge, \mathrm{w})$.

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  - $\mathrm{OBDD}(\wedge, \mathrm{w})$ does not simulate $\mathrm{Res}(O(\log n))$.
- [Buss, I., Knop, Sokolov, 2018] Reordering rule makes proof systems stronger.
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OBDD($\wedge$, weakening)-proofs

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- [Atserias, Kolaitis, Vardi, 2004] There is an order π s.t. all $\pi - \mathrm{OBDD}(\wedge, \mathrm{w})$ proofs of Clique-Coloring are of exp. size.
- [Krajicek, 2008] $2^{n^{\Omega(1)}}$-lower bound for dag-like $\mathrm{OBDD}(\wedge, \mathrm{w})$-proofs:
  - $\varphi(x)$ is a formula hard for one order π;
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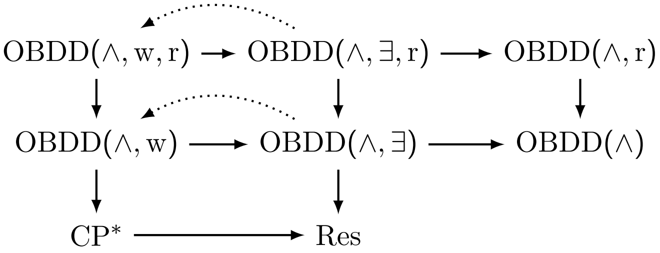
OBDD picture



- If there is a path consisting of solid (straight) edges from Π_1 and Π_2, then Π_1 simulates Π_2.
- If there is a path from Π_1 to Π_2, but every such path contains a dotted (arched) edge, then it is **open**, whether Π_1 simulates Π_2.
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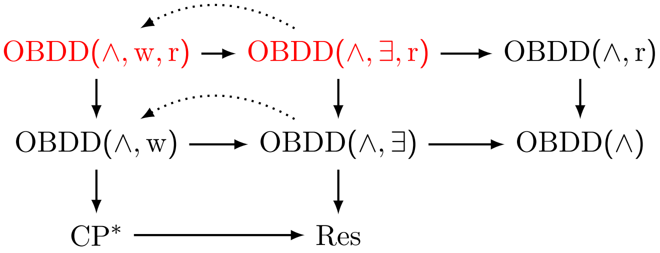

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Hardness of automation

Theorem. [I., Riazanov, 2022] There exists a polytime function $\mathcal{R}$ mapping CNF formulas to CNF formulas: for any 3-CNF ϕ with n variables

- if $\phi \in \mathrm{SAT}$, then $\mathcal{R}(\phi)$ has a resolution refutation of size at most n^{α};
- if $\phi \in \mathrm{UNSAT}$, then any $\mathrm{OBDD}(\wedge, \mathrm{w})$ refutation of $\mathcal{R}(\phi)$ has size $2^{\Omega(n)}$.

Corollary. It is NP-hard to automate $\mathrm{OBDD}(\wedge, \mathrm{w})$ and $\mathrm{OBDD}(\wedge, \exists)$.

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1-BP($\wedge$)

- 1-BP($\wedge$) has short refutations for formulas based on bipartite graphs: PHP, Tseitin formulas on bipartite graphs, etc.
- [Buss, I., Knop, Riazanov, Sokolov, 2021] Lower bound for 1-BP($\wedge$):
  - $\mathrm{PM}(G)$: graph $G(V, E)$ has a perfect matching:
    - Every $v \in V$ is covered: $\bigvee_{v \in e} x_e$
    - v is not covered twice.
  - **Theorem.** If G is good enough expander, then $\mathrm{PM}(G)$ and Tseitin(G) require 1-BP($\wedge$) of size $2^{\Omega(n)}$.
  - **Proof idea:** Consider a moment, when 1-BP contains $\theta(|V|)$ clauses of the first type, then prove that the size of 1-BP representation is exponential.
- Lower bound also holds for 1-NBP($\wedge$).
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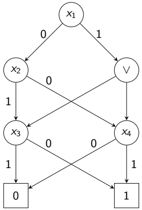

OBDD-based SAT algorithms

Input: CNF formula ϕ

1. Choose order π, D^π. Initially $D \equiv 1$.
2. $S :=$ {clauses of ϕ}.
3. While $S \neq \emptyset$ apply the following operations:
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Hard formulas for 1-$\mathrm{NBP}(\wedge, \exists)$ SAT algorithms

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  - Formula encodes that $x \in C$.
- [I., 2021] Hard unsatisfiable formulas:
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2. Separate $\mathrm{OBDD}(\wedge, \exists)$ and $\mathrm{OBDD}(\wedge, \mathrm{w})$. Separation candidate: Clique Coloring principle.
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5. Separate dag-like and tree-like $\mathrm{OBDD}(\wedge)$.
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8. Is $\mathrm{OBDD}(\wedge)$ automatable?

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2. Separate $\mathrm{OBDD}(\wedge, \exists)$ and $\mathrm{OBDD}(\wedge, \mathrm{w})$. Separation candidate: Clique Coloring principle.
3. Prove lower bound for $\mathrm{OBDD}(\wedge, \mathrm{w}, \mathrm{r})$.
4. Does AC_0-Frege simulate $\mathrm{OBDD}(\wedge)$? Does resolution quasi-polynomially simulate $\mathrm{OBDD}(\wedge)$?
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